

---

# Index

- o* notation, 231
  
- absolute value, 7, 157
- action, of a group, 150, 152
  - examples, 153–157
  - free and discontinuous, 151
- algorithm, 38, 155, 186
- alternating group, 257
- angle, 73, 105, 114
  - external, 115
  - internal, 116
- antipodal, 24, 52
  - map, 52, 154
- arc, 58
- argument (of complex number), 9
- Artin’s criterion, 91, 94, 111
- Atiyah, Sir Michael, 169, 180
- Atkinson’s theorem, 127, 176
  
- Baire category theorem, 247
- ball, 33, 49, 66, 204
- basepoint, 20, 139, 142
  - and winding number, 47
- basis, 7, 39, 81, 122, 123, 161, 185
  - right-handed, 8, 34, 119
- best linear approximation, 232
- Bolzano-Weierstrass theorem, 210
- Bombelli, Rafael, 42, 255
- Borsuk-Ulam theorem, 52
- Bott periodicity theorem, 175
  
- boundary, 55, 59, 71, 82, 90, 165, 237
  - in homology theory, 90
- bounded, 57, 99
  - linear map, 130, 246
  - subset of a metric space, 206, 209
- bouquet of circles, 159
- Brouwer fixed-point theorem, 50, 112, 143, 172
- bubble argument, 36, 44
  
- calculus, xi, 86, 233
  - fundamental theorem of, 78, 87, 105
  - symbolic, 132
- Calkin algebra, 127, 176
- Carathéodory, Constantin, 66
- Cardano, Gerolamo, 42
- Cauchy sequence, 125, 211, 245
- Cauchy’s theorem, 94
- Cauchy-Riemann equations, 65, 85
- Cauchy-Schwarz inequality, 195
- Cayley graph, 157
- cell, 34, 55
- chain, 88
- chain rule, 99, 234
- Clairaut’s theorem, 80, 96, 235
- closed, 205, 208, 209
  - form, *see also* form, closed

- subspace of Hilbert space, 123, 131
- closed graph lemma, 123, 247
- codimension, 122, 136, 189
- compact
  - covering, 211
  - operator on Hilbert space, 247
  - sequentially, 209
- complete, 211, 230, 240
- component
  - path, 16, 33, 55, 124
  - unbounded, 34
- concatenation, 16, 32, 62, 140
- conformal mapping, 64
- conjugate, 7
- connected, 17, 163
  - locally, 149
  - path, 15, 16, 22
  - simply, *see also* simply connected
- continuous, 206
  - in terms of open sets, 207
  - uniformly, 213
- contractible, 26, 58, 170, 176
- conullity, 122
- corank, 122
- coset, 160, 254
- covering map, 144, 152, 163
- curvature, 105, 106
  - form, 106
- cycle, 89, 91, 111
- de Rham cohomology, 72, 81, 99
- degree
  - of map  $S^1 \rightarrow S^1$ , 51
  - of map  $S^n \rightarrow S^n$ , 171
  - of polynomial, 42
  - of singularity, 109
- dense, 38, 131, 209
- derivative, 232
  - directional, 233
- determinant, 135, 199–202, 257
- diffeomorphism, 98
- differential form, *see also* form
- differentiation
  - of a path, 38
  - of function between vector spaces, 232
- dimension, 17, 98, 121, 184–188
- directional derivative, 74, 233
- discrete, 16, 144, 205
- Dold, Albrecht, 67
- dual space, 74, 192–194, 233
- eigenvalue, 14, 46, 68
- Eilenberg’s criterion, 56, 63
- Eilenberg-MacLane space, 170
- entire function, 8, 99
- equivalence relation, 15, 19, 89, 154, 250
- Euclidean space, 15, 204
- Euler characteristic, 113, 114
- even map, 51
- exact sequence, 137, 174, 192
- exponential function, 6–12, 27, 30, 56, 94, 136, 145, 146
  - illustrated, 10
- exponential law, 154
- fiber, 144, 176
- fibration, 136, 173–174
- final point, 15
- form, 74
  - closed, 80, 81, 85, 90, 111
  - and integration, 82
  - exact, 80
  - gradient, 74, 75
  - pullback, 75, 80
- Fredholm operator, 123, 133, 176
- free group, 154, 157
- Freudenthal suspension theorem, 171
- functorial, 76, 99, 142, 149, 171, 194
- fundamental group, 139–167
- fundamental theorem of algebra, 42, 44, 100
- Garcia, Jerry, 5
- general position, 24, 170, 223
- Ghomi, Mohammad, 118
- gluing lemma, 16, 149, 214
- grid, 91
- group, 88, 134, 140, 169, 249–257
  - see* fundamental group, 257
- ham sandwich theorem, 53
- Hamilton, William Rowan, 13, 161

- Hamming metric, 204
- Hardy space, 131
- Heine-Borel property, 211
- Hilbert space, 178, 239–247
- holomorphic, 64, 84
- homeomorphism, 54, 207, 213
- homology, 87–94, 100
- homomorphism, 99, 132, 254–257
  - connecting, 174
  - induced, 142
- homotopy, 19, 28, 29, 83, 100, 139, 146, 160, 170
  - of loops, 22–25
- homotopy class, 19
- homotopy equivalence, 143
- homotopy group, higher, 169
- homotopy lifting theorem, 146
- Hopf index theorem, 114
- Hopf map, 172
- Hopf, Heinz, 102, 114, 161
- image
  - of a chain, 88
  - of a path, 33
  - of linear map, 122, 188
- increment formula, 233
- index
  - Fredholm, 123, 124
  - of a subgroup, 254
  - of singularity, 109
- initial point, 15
- inner product, 194–197, 239
- integration
  - along chain, 89
  - along path, *see also* path, integration along
  - by parts, 98
  - by substitution, 76
- intermediate value theorem, 4, 16
- intersection, 73
- intersection number, 36, 39
- invariance of domain, 63
- invertibility criterion, 202
- isometry, 151, 208
- Jordan curve, 54, 102
- Jordan curve theorem, 54, 56, 94
  - proof of, 59–63
- Jordan domain, 64
- kernel, 122, 188
- Klein bottle, 165
- Koebe, Paul, 65
- Kuiper, Nicolaas, 176
- Lagrange’s theorem, 254
- Lebesgue number, 147, 211
- lemonade, xi, 81, 115
- lens space, 165
- lifting, 30, 136, 143–150, 153, 170
- Liouville’s theorem, 100
- local homeomorphism, 163, 167
- logarithm, 12, 28
- loop, 4, 30, 31, 54, 83, 133, 139
  - monotonic, 102
  - polygonal, 36
  - regular, *see also* path, regular smooth, 38, 101
- loop space, 20, 31, 174
- lovers and haters, 1–4, 47, 59
- Maehara, Ryuji, 60
- map, 27, 206
- measure theory, 53, 68, 243
- measure zero, 226
- metric space, 55, 150, 203–206, 240
- Möbius band, 165
- modest, 147–149
- modulus, 7, 9
- multiplication operator, 130
- multiplicity, 43, 88
- no-retraction theorem, 49, 172
- norm, 196, 204, 229–231
- Novikov, Pyotr, 157
- nullhomologous, 90, 100
- nullity, 122, 191
- number of sheets, 144, 160
- odd map, 51
- open, 125, 204
  - cover, 211
- operator, 246
- orbit, 151
- orthogonal, 243
- paradoxical decomposition, 166
- parallelogram law, 243

- parentheses, convention for shape  
     of, 74, 232  
 parity theorem, 256  
 path, 3, 15, 19, 20, 29, 32, 38, 56,  
     128, 135, 206  
     integration along, 77–79  
     piecewise straight, 25  
     polygonal, 36, 79  
     regular, 38, 101, 107  
     smooth  
         piecewise, 79  
         unit speed, 105  
 path connected, *see also* connected  
 path space, 20  
     fibration, 174  
 Peano, Guiseppe, 17  
 Perelman, Grisha, 66  
 Perron-Frobenius theorem, 68  
 pole, 6, 45  
 positive  
     direction of rotation, 5, 10  
     square root, 12  
 projection, 131, 147, 184  
 projection theorem, 128, 244  
 projective plane, 154  
 Ptolemy, 13  
 punctured plane, 23, 27, 174  
 Pythagoras's theorem, 243  
  
 quadratic map, 162  
  
 radial retraction, 32, 158, 164  
 rank, 122, 191  
 rank-nullity theorem, 122, 191  
 rational function, 45  
 reparameterization, 19, 140  
     and winding number, 47  
     smooth, 78, 105, 106  
 representation theorem, 196  
 reverse, 16, 89, 141  
 Riemann mapping theorem, 65, 95  
 rotation number, 101, 110  
 Rouché's theorem, 28, 45, 110  
 row and column operations, 126  
  
 Sard's theorem, 41, 171  
 Schoenflies theorem, 64  
  
 secant map, 103, 117  
 second derivative, 235  
 sequence, 208  
 short straight section, 34  
 sign, of a permutation, 199, 256  
 simply connected, 22, 99, 136, 146  
 singularity  
     index of, 109  
     of vector field, 108  
 slit plane, 12, 22, 28  
 smooth, 38, 162, 235  
 space-filling curve, 17–18, 24, 26, 98  
 sphere, 23, 49  
 standard operating procedure, 86,  
     133  
 star-shaped, 22  
 stereographic projection, 23, 120  
 Stone-Weierstrass theorem, 38, 46,  
     131, 219  
 subspace  
     complementary, 184  
 supersymmetry, 179  
 surjection, 143  
 suspension, 171  
 symbolic calculus, 132  
 symmetric group, 151, 199, 252  
  
 theorem of the turning tangent, 103  
 Tietze extension theorem, 57, 67,  
     221  
 Toeplitz index theorem, 133, 179  
     matrix version, 134  
 Toeplitz operator, 129–136, 178  
     definition of, 132  
 torus, 153  
 transverse, 36, 41, 171  
 triangle inequality, 152, 196, 203  
 trigonometric polynomial, 130  
 trivializing cover, 145  
  
 uncrossing, 118  
 uniform distance, 214  
 unilateral shift, 124  
 unitization, 32  
  
 vector field, 108

- 
- Whitney-Graustein theorem, 117  
winding number, 5, 29–46, 51, 73,  
91, 172  
definition of, 30
- word, 154  
problem, 156  
reduced, 155