
Contents

Foreword: MASS at Penn State University	vii
Preface	ix
Chapter 1. Graph Ramsey theory	1
§1.1. The basic setting	1
§1.2. The basics of graph theory	4
§1.3. Ramsey's theorem for graphs	14
§1.4. Ramsey numbers and the probabilistic method	21
§1.5. Turán's theorem	31
§1.6. The finite Ramsey theorem	34
Chapter 2. Infinite Ramsey theory	41
§2.1. The infinite Ramsey theorem	41
§2.2. König's lemma and compactness	43
§2.3. Some topology	50
§2.4. Ordinals, well-orderings, and the axiom of choice	55
§2.5. Cardinality and cardinal numbers	64
§2.6. Ramsey theorems for uncountable cardinals	70
§2.7. Large cardinals and Ramsey cardinals	80

Chapter 3. Growth of Ramsey functions	85
§3.1. Van der Waerden's theorem	85
§3.2. Growth of van der Waerden bounds	98
§3.3. Hierarchies of growth	105
§3.4. The Hales-Jewett theorem	113
§3.5. A really fast-growing Ramsey function	123
Chapter 4. Metamathematics	129
§4.1. Proof and truth	129
§4.2. Non-standard models of Peano arithmetic	145
§4.3. Ramsey theory in Peano arithmetic	152
§4.4. Incompleteness	159
§4.5. Indiscernibles	171
§4.6. Diagonal indiscernibles via Ramsey theory	182
§4.7. The Paris-Harrington theorem	188
§4.8. More incompleteness	193
Bibliography	199
Notation	203
Index	205