

Preface

Over the last ten years, the theory of Bergman spaces has undergone a remarkable metamorphosis. In a series of major advances, central problems once considered intractable were solved, and a rich theory has emerged. Although progress continues, the time seems ripe for a full and unified account of the subject, weaving old and new results together.

The modern subject of Bergman spaces is a blend of complex function theory with functional analysis and operator theory. It comes in contact with harmonic analysis, approximation theory, hyperbolic geometry, potential theory, and partial differential equations. Our aim has been to develop background material and make the subject accessible to a broad segment of the mathematical community. We hope the book will prove useful not only as a reference for research workers, but as a text for graduate students.

In fact, the book evolved from a rough set of notes prepared for graduate students in a two-week course that one of us gave in 1996 at the Norwegian University of Science and Technology in Trondheim, in conjunction with a conference on Bergman spaces supported by the Research Council of Norway. Since that time we have used successive versions of the manuscript in graduate courses we taught at the University of Michigan (1998), Washington University in St. Louis (1999), and San Francisco State University (2001). The last course was supported by the NSF CIRE (Collaborative to Integrate Research and Education) program. The students in all of these courses were enthusiastic, and their perceptive remarks on the manuscript often led to substantial improvements.

In striving for clear presentations of material, we have had the benefit of expert advice from many friends and colleagues. We are most grateful to Kristian Seip for guiding us to a self-contained account of his deep results on interpolation and sampling. Harold Shapiro showed us an elegant way to develop the biharmonic Green function and helped with other constructions. Dmitry Khavinson fielded a steady barrage of technical questions and offered many useful suggestions on the manuscript. Sheldon Axler made a careful reading of several chapters and gave valuable criticism. Mathematical help of various sorts came also from Marcin Bownik, Brent Carswell, Eric Hayashi, Håkan Hedenmalm, Anton Kim, John McCarthy, Maria Nowak, Stefan Richter, Richard Rochberg, Joel Shapiro, Michael Stessin, Carl Sundberg, James Tung, Dror Varolin, Dragan Vukotić, Rachel Weir, and Kehe Zhu. Special thanks go to Joel Shapiro for permission to

base our treatment of cyclic singular inner functions (Section 8.2) on his unpublished notes. Christopher Hammond read large portions of the manuscript with an eagle eye and spotted a number of misprints, minor errors, and obscurities. Anders Björn also made helpful remarks. We want to express our sincere appreciation to all of these people, and others whose names we may have overlooked, for helping to improve this book. Any defects that remain, however, are the authors' responsibility.

We also had the benefit of the earlier book *Theory of Bergman Spaces* by Håkan Hedenmalm, Boris Korenblum, and Kehe Zhu (Springer-Verlag, 2000), which served as a useful reference in our approach to several topics. As may be expected, the two books have considerable overlap, but ours develops more of the prerequisite material. It also treats topics not discussed in the earlier book, and treats some of the same topics in different ways. A few results appear here for the first time. On the other hand, the book of Hedenmalm, Korenblum, and Zhu contains extensive discussions of several topics barely touched upon in our book, such as invertible noncyclic functions and logarithmically subharmonic weights.

Our book is essentially self-contained. It should be accessible to advanced graduate students who have studied basic complex function theory, measure theory, and functional analysis. Prior knowledge of Hardy spaces is helpful, since that theory often serves as a model for Bergman spaces, but the main facts about Hardy spaces are reviewed in two "crash courses" early in the book and later as motivation for corresponding topics in Bergman spaces. A few Hardy space results are actually needed for the theoretical development of Bergman spaces, and proofs are given.

Most of the writing was carried out during summers together in Ann Arbor, where the University of Michigan provided excellent facilities for our work. Thanks also go to the Ann Arbor Diamondbacks, who were an extra incentive for the second-named author to return to Michigan every summer.

Over the last decade, the American Mathematical Society held several Special Sessions on Bergman spaces at national and regional meetings, and sponsored a week-long research conference at Mt. Holyoke College in the summer of 1994. That summer conference, in particular, did much to stimulate further research in the field. We were therefore especially pleased when the AMS agreed to publish our book. We are grateful to Sergei Gelfand of the AMS for his initial vision that encouraged us not to settle for a revised set of lecture notes, but to do the extra work needed to produce a full expository account of the subject. He showed remarkable patience with the slow pace of the resulting project, but pushed us to finish when the end was in sight and helped with the technical aspects of production. We hope our book may be judged a worthy successor to the classic book by Stefan Bergman, which appeared in the same AMS series many years ago.

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