# Preface

This work began as a research paper intended to show how the convergence of nonlinear semigroups associated with a sequence of Markov processes implied the large deviation principle for the sequence. We expected the result to be of little utility for specific applications, since classical convergence results for nonlinear semigroups involve hypotheses that are very difficult to verify, at least using classical methods. We should have recognized at the beginning that the modern theory of viscosity solutions provides the tools needed to overcome the classical difficulties. Once we did recognize that convergence of the nonlinear semigroups could be verified, the method evolved into a unified treatment of large deviation results for Markov processes, and the research "paper" steadily grew into the current volume.

There are many approaches to large deviations for Markov processes, but this book focuses on just one. Our general title reflects both the presentation in Part 1 of the theory of large deviations based on the large deviation analogue of the compactness theory for weak convergence, material that is the foundation of several of the approaches, and by the generality of the semigroup methods for Markov processes.

The goal of Part 2 is to develop an approach for proving large deviations, in the context of metric-space-valued Markov processes, using convergence of generators in much the same spirit as for weak convergence (e.g. Ethier and Kurtz [**36**]). This approach complements the usual method that relies on asymptotic estimates obtained through Girsanov transformations.

The usefulness of the method is best illustrated through examples, and Part 3 contains a range of concrete examples.

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