Preface and Apology

These lecture notes are a compilation of surveys of the topics that were presented in a series of talks at Alcalá, Spain, August 31 – September 5, 2006, by János Pach and Micha Sharir. To a large extent, these surveys are adapted from earlier papers written by the speakers and their collaborators. In their present form, the notes aptly describe both the history and the state of the art of these topics.

The notes are arranged in an order that roughly parallels the order of the talks. Chapter 1 describes the beginnings of combinatorial geometry: Starting with Sylvester's problem on the existence of "ordinary lines," we introduce a number of exciting problems on incidences between points and lines in the plane and in space. This chapter uses the material in Pach, Pinchasi, and Sharir [**588, 589**]. In Chapter 2 we survey many aspects of the theory of *arrangements of surfaces* in higher dimensions. It is adapted from Agarwal and Sharir [**51**]. Readers that have some familiarity with the basic theory of arrangements can start their reading on this topic from this chapter, while those that are complete novices may find it useful to look first at Chapter 3, which studies arrangements of *curves* in the plane, with special emphasis on *Davenport–Schinzel sequences* and the major role they play in the theory of arrangements. This chapter is adapted from Agarwal and Sharir [**52**].

Chapter 4 covers the topic of incidences between points and curves and its many relatives, where a surge of activity has taken place in the past decade. It is adapted from two similar surveys by Pach and Sharir [**600, 601**]. The study of combinatorial and topological properties of planar arrangements of curves has become a separate discipline in discrete and computational geometry, under the name of *Graph Drawing.* Some basic aspects of this emerging discipline are discussed in Chapter 5, which is based on the survey by Pach [**583**]. Some classical questions of Erdős on repeated interpoint distances in a finite point set can be reformulated as problems on the maximum number of incidences between points and circles, spheres, etc. In fact, these questions motivated and strongly influenced the early development of the theory of incidences a quarter of a century ago and they led to the discovery of powerful new combinatorial and topological tools. Many of Erdős's questions can be naturally generalized to problems on larger repeated subpatterns in finite point sets. Based on Brass and Pach [**175**], in Chapter 6 we outline some of the most challenging open problems of this kind, whose solution would have interesting consequences in *pattern matching* and recognition.

Chapter 7 treats the special topic of lines in three-dimensional space, which is a nice application (or showpiece, if you will) of the general theory of arrangements on one hand, and shows up in a variety of only loosely related topics, ranging from *ray shooting* and *hidden surface removal* in computer graphics to geometric *transversal theory.* This chapter partially builds upon a somewhat old paper by Chazelle, Edelsbrunner, Guibas, Sharir, and Stolfi [**220**], but its second half is new,

and presents (some of) the recent developments. Some combinatorial properties of arrangements of spheres, boxes, etc., are discussed in Chapter 8. They raise difficult questions on the *chromatic numbers* and other similar parameters of certain geometrically defined graphs and hypergraphs, with possible applications to *frequency allocation* in cellular telephone networks. Here we borrowed some material from Pach, Tardos, and Tóth [**608**].

An old and rich area of applications of Davenport–Schinzel sequences and the theory of geometric arrangements is *motion planning*. Starting with Sam Loyd's coin puzzles, in Chapter 9 we discuss a number of problems that can be regarded as discrete variants of the "piano movers' problem" on graphs and grids. Some of the results have applications to the reconfiguration of metamorphic *robotic systems*. This chapter is based on recent joint papers with Bereg, Călinescu, and Dumitrescu [**138, 190, 280**].

While we have made our best attempts to make these notes comprehensive, they are not at the level of a polished monograph, nor do they provide full coverage of all the relevant recent results and developments. It is our hope that they be a useful source of reference to the rich and extensive theory presented at the Alcalá series of talks.

Apart from those friends and coauthors whose names were mentioned above, we freely borrowed from joint work with D. Pálvölgyi and R. Wenger. We are extremely grateful to all of them for the enjoyable and fruitful collaboration and for their kind permission to reproduce their ideas and "plagiarize" their words. Our thanks are also due to Sergei Gelfand, Gabriel Nivasch, and Deniz Sarioz for their invaluable help in finalizing the manuscript.

János Pach (New York and Budapest)
Micha Sharir (Tel Aviv and New York)