

# CONTENTS

PREFACE TO THE SECOND EDITION . . . . .	ix
PREFACE TO THE FIRST EDITION . . . . .	xi
ABOUT THIS BOOK . . . . .	xiii
INTRODUCTION . . . . .	1
CHAPTER 1. THE BASIC CLASSICAL NOTIONS . . . . .	9
1A. Perfect Polish spaces . . . . .	9
1B. The Borel pointclasses of finite order . . . . .	13
1C. Computing with relations; closure properties . . . . .	18
1D. Parametrization and hierarchy theorems . . . . .	26
1E. The projective sets . . . . .	29
1F. Countable operations . . . . .	33
1G. Borel functions and isomorphisms . . . . .	37
1H. Historical and other remarks . . . . .	46
CHAPTER 2. $\kappa$ -SUSLIN AND $\lambda$ -BOREL . . . . .	49
2A. The Cantor-Bendixson Theorem . . . . .	50
2B. $\kappa$ -Suslin sets . . . . .	51
2C. Trees and the Perfect Set Theorem . . . . .	57
2D. Wellfounded trees . . . . .	62
2E. The Suslin Theorem . . . . .	65
2F. Inductive analysis of projections of trees . . . . .	70
2G. The Kunen-Martin Theorem . . . . .	74
2H. Category and measure . . . . .	79
2I. Historical remarks . . . . .	85
CHAPTER 3. BASIC NOTIONS OF THE EFFECTIVE THEORY . . . . .	87
3A. Recursive functions on the integers . . . . .	89
3B. Recursive presentations . . . . .	96
3C. Semirecursive pointsets . . . . .	101
3D. Recursive and $\Gamma$ -recursive functions . . . . .	110
3E. The Kleene pointclasses . . . . .	118
3F. Universal sets for the Kleene pointclasses . . . . .	125
3G. Partial functions and the substitution property . . . . .	130
3H. Codings, uniformity and good parametrizations . . . . .	135
3I. Effective theory on arbitrary (perfect) Polish spaces . . . . .	141

3J.	Historical remarks .....	142
CHAPTER 4.	STRUCTURE THEORY FOR POINTCLASSES .....	145
4A.	The basic representation theorem for $\Pi_1^1$ sets .....	145
4B.	The prewellordering property .....	152
4C.	Spector pointclasses .....	158
4D.	The parametrization theorem for $\Delta \cap \mathcal{X}$ .....	165
4E.	The uniformization theorem for $\Pi_1^1, \Sigma_2^1$ .....	173
4F.	Additional results about $\Pi_1^1$ .....	184
4G.	Historical remarks .....	202
CHAPTER 5.	THE CONSTRUCTIBLE UNIVERSE .....	207
5A.	Descriptive set theory in $L$ .....	208
5B.	Independence results obtained by the method of forcing .....	214
5C.	Historical remarks .....	215
CHAPTER 6.	THE PLAYFUL UNIVERSE .....	217
6A.	Infinite games of perfect information .....	218
6B.	The First Periodicity Theorem .....	229
6C.	The Second Periodicity Theorem; uniformization .....	235
6D.	The game quantifier $\mathfrak{D}$ .....	244
6E.	The Third Periodicity Theorem .....	254
6F.	The determinacy of Borel sets .....	272
6G.	Measurable cardinals .....	280
6H.	Historical remarks .....	290
CHAPTER 7.	THE RECURSION THEOREM .....	293
7A.	Recursion in a $\Sigma^*$ -pointclass .....	293
7B.	The Suslin-Kleene Theorem .....	298
7C.	Inductive definability .....	309
7D.	The completely playful universe .....	323
7E.	Historical remarks .....	339
7F.	Results which depend on the Axiom of Choice .....	341
CHAPTER 8.	METAMATHEMATICS .....	353
8A.	Structures and languages .....	355
8B.	Elementary definability .....	365
8C.	Definability in the universe of sets .....	371
8D.	Gödel's universe of constructible sets .....	381
8E.	Absoluteness .....	390
8F.	The basic facts about $L$ .....	401
8G.	Regularity results and inner models .....	416
8H.	On the theory of indiscernibles .....	446
8I.	Some remarks about strong hypotheses .....	468
8J.	Historical remarks .....	473
	THE AXIOMATICS OF POINTCLASSES .....	475
	REFERENCES .....	477
	INDEX .....	491