

Preface

This book aims to provide a systematic account for some recent progress on the large deviations arising from the area of sample path intersections, including calculation of the tail probabilities of the intersection local times, the ranges and the intersections of the ranges of random walks and Brownian motions. The phrase “related topics” appearing in the title of the book mainly refers to the weak law and the law of the iterated logarithm for these models. The former is the reason for certain forms of large deviations known as moderate deviations; while the latter appears as an application of the moderate deviations.

Quantities measuring the amount of self-intersection of a random walk, or of mutual intersection of several independent random walks have been studied intensively for more than twenty years; see *e.g.* [57], [59], [124], [125], [116], [22], [131], [86], [135][136], [17], [90], [11], [10], [114]. This research is often motivated by the role that these quantities play in renormalization group methods for quantum field theory (see *e.g.* [78], [51], [52], [64]); our understanding of polymer models (see *e.g.* [134], [19],[96], [98] [162], [165], [166],[167],[63], [106], [21], [94], [93]); or the analysis of stochastic processes in random environments (see *e.g.* [107], [111],[43], [44] [82], [95], [4], [42] [79], [83]).

Sample path intersection is also an important subject within the probability field. It has been known ([48], [138], [50]) that sample path intersections have a deep link to the problems of cover times and thick points through tree-encoding techniques. In addition, it is impossible to write a book on sample path intersection without mentioning the influential work led by Lawler, Schramm and Werner ([118], [119], [120], [117]) on the famous intersection exponent problem and on other Brownian sample path properties in connection to the Stochastic Loewner Evolution, which counts as one of the most exciting developments made in the fields of probability in recent years.

Contrary to the behavior patterns investigated by Lawler, Schramm and Werner, where the sample paths avoid each other and are loop-free, most of this book is concerned with the probability that the random walks and Brownian motions intersect each other or themselves with extreme intensity. When these probabilities decay at exponential rates, the problem falls into the category of large deviations. In recent years, there has been some substantial input about the new tools and new ideas for this subject. The list includes the method of high moment asymptotics, sub-additivity created by moment inequality, and the probability in Banach space combined with the Feynman-Kac formula. Correspondent to the progress in methodology, established theorems have been accumulated into a rather complete

picture of this field. These developments make it desirable to write a monograph on this subject which has not been adequately exposed in a systematic way.

This book was developed from the lecture notes of a year-long graduate course at the University of Tennessee. Making it accessible to non-experts with only basic knowledge of stochastic processes and functional analysis has been one of my guidelines in writing it. To make it reasonably self-contained, I added Chapter 1 for the general theory of large deviations. Most of the theorems listed in this chapter are not always easy to find in literature. In addition, a few exercises are included in the “Notes and comments” section in each chapter, an effort to promote active reading. Some of them appear as extensions of, or alternative solutions to the main theorems addressed in the chapter. Others are not very closely related to the main results on the topic, such as the exercises concerning small ball probabilities, but are linked to our context by sharing similar ideas and treatments. The challenging exercises are marked with the word “hard”. The mainspring of the book does not logically depend on the results claimed in the exercises. Consequently, skipping any exercise does not compromise understanding the book.

The topics and results included in the book do reflect my taste and my involvement on the subject. The “Notes and comments” section at the end of each chapter is part of the effort to counterbalance the resulted partiality. Some relevant works not included in the other sections may appear here. In spite of that, I would like to apologize in advance for any possible inaccuracy of historic perspective appearing in the book.

In the process of investigating the subject and writing the book, I benefitted from the help of several people. It is my great pleasure to acknowledge the contributions, which appear throughout the whole book, made by my collaborators R. Bass, W. Li, P. Mörters and J. Rosen in the course of several year’s collaboration. I would like to express my special thanks to D. Khoshnevisan, from whom I learned for the first time the story about intersection local times. I thank A. Dembo, J. Denzler, A. Dorogovtsev, B. Duplantier, X. B. Feng, S. Kwapień, J. Rosinski, A. Freire, J-F. Le Gall, D. S. Wu, M. Yor for discussion, information, and encouragement. I appreciate the comments from the students who attended a course based on a preliminary version of this book, Z. Li, J. Grieves and F. Xing in particular, whose comments and suggestions resulted in a considerable reduction of errors. I am grateful to M. Saum for his support in resolving the difficulties I encountered in using latex.

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