Preface

This book can be considered as the second volume of the author's monograph "Mathematical Scattering Theory (General Theory)" [I]. It is oriented to applications to differential operators, primarily to the Schrödinger operator. A necessary background from [I] is collected (but the proofs are of course not repeated) in Chapter 0. Therefore it is presumably possible to read this book independently of [I].

Everything said in the preface to [I] pertains also to this book. In particular, we proceed again from the stationary approach. Its main advantage is that, simultaneously with proofs of various facts, the stationary approach gives formula representations for the basic objects of the theory. Along with wave operators, we also consider properties of the scattering matrix, the spectral shift function, the scattering cross section, etc.

A consistent use of the stationary approach as well as the choice of concrete material distinguishes this book from others such as the third volume of the course of M. Reed and B. Simon [**43**]. The latter course has become a desktop copy for many, in particular, for the author of the present book. However, in view of the broad compass of material, the course [**43**] was necessarily written in encyclopedic style and apparently cannot replace a systematic exposition of the theory. Hopefully, vol. 3 of [**43**] and this book can be considered as complementary to one another.

There are two different trends in scattering theory for differential operators. The first one relies on the abstract scattering theory. The second one is almost independent of it. In this approach the abstract theory is replaced by a concrete investigation of the corresponding differential equation. In this book we present both of these trends.

The first of them illustrates basic theorems of [I]. Thus, Chapters 1 and 2 are devoted to applications of the smooth method. Of course the abstract results of [I] should be supplemented by some analytic tools, such as the Sobolev trace theorem. The smooth method works well for perturbations of differential operators with constant coefficients. In Chapter 3 applications of the trace class method are discussed. The main advantage of this method is that it does not require an explicit spectral analysis of an "unperturbed" operator.

Other chapters are much less dependent on [I]. Chapters 4 and 5 are devoted to the one-dimensional problem (on the half-axis and the entire axis, respectively) which is a touchstone for the multidimensional case because specific methods of ordinary differential equations can be used here.

In the following chapters we return to the multidimensional problem and discuss different analytic methods appropriate to differential operators. In particular, in Chapter 6 scattering theory is formulated in terms of solutions of the Schrödinger equation satisfying some "boundary conditions" (radiation conditions) at infinity.

High- and low-energy asymptotics of the Green function (the resolvent kernel) and of related objects are discussed in Chapter 7. Chapter 8 is devoted to a study of the scattering matrix and of the scattering cross section. Here some asymptotic methods, such as the ray expansion and eikonal expansion, are also discussed.

As an example of a useful interaction of abstract and analytic methods, we mention the theory of the spectral shift function. Abstract results are illustrated in §3.8. However, specific properties of this function are studied by concrete methods in §4.5, §5.3 and in Chapter 9. Here perturbation determinants are also discussed and trace identities are derived.

Note that Chapters 1 and 3 and large parts of Chapters 4 and 5 contain essentially a "necessary minimum" on scattering theory, whereas the other chapters are of a slightly more special nature.

The book is mainly devoted to a study of perturbations by differential operators with short-range coefficients. Nevertheless, basic results on long-range scattering, in particular, properties of the scattering matrix, can be found in Chapter 10.

We mention that the recent progress in scattering theory is to a large extent related to multiparticle systems. This very interesting and difficult problem is discussed in [**16**] and [**61**].

Similarly to [I], in working on the book the author has tried to resolve two opposite problems. The first of them is a systematic exposition of the material starting from the general background of [I]. The second problem is the exposition of a number of topics to a degree of completeness which might possibly be of interest to experts in spectral theory. We have also tried to fill in numerous gaps present in monographic literature. This pertains especially to the exposition of works of Russian and, in particular, Saint Petersburg mathematicians. Compared to [I], the author's tastes are also more thoroughly represented here. As a whole the book is oriented toward a reader (for example, a graduate student in mathematical physics) interested in a deeper study of scattering theory.

In references we use the "three-stage" enumeration of formulas and theorems and the "two-stage" enumeration of sections. However, the first number is omitted within a chapter.

This book is based on the graduate courses taught by the author several times in Saint-Petersburg and Rennes Universities.

The concept and structure of the entire book, as well as many specific questions, were discussed with the author's teacher M. Sh. Birman. To a large extent, mathematical tastes of the author were influenced by L. D. Faddeev. The author is deeply grateful to M. Sh. Birman and L. D. Faddeev. Numerous discussions with P. Deift, A. B. Pushnitski, G. Raikov and M. Z. Solomyak are also gratefully acknowledged.

Interdependence of chapters