

Preface

Dynamical systems theory has been a very active area of research in mathematics and cognate fields for many years, but most of the results that have been obtained focus almost exclusively on autonomous dynamical systems. There have, of course, been investigations of nonautonomous differential equations, that is with time-dependent vector fields, during this time, but it is only in the recent decade that a theory of nonautonomous dynamical systems has emerged synergizing parallel developments on time-dependent differential equations, control systems and random dynamical systems. There are now abstract formulations of nonautonomous dynamical systems as two-parameter semi-groups or processes and as skew product flows as well as new concepts of nonautonomous attractors, in particular, pullback attractors.

This development is presented in this book for graduate students and others with a general background in dynamical systems and differential equations. The choice of topics and applications covered, especially in the later chapters, reflects the interests of the authors, but nevertheless provides a broad overview of important developments on the subject.

There are fifteen chapters and an appendix. The first chapter briefly reviews the theory of autonomous dynamical systems from the perspective of what is needed later rather than attempting to be comprehensive in itself. The process and skew product flow formalism of nonautonomous dynamical systems are introduced in the second chapter and the various concepts of nonautonomous attractors are presented and compared in the third chapter. These two chapters are essential reading for everything that is to follow. The remaining chapters can be read more or less independently of each other, except the fourth, fifth and sixth chapters on Morse decompositions, linear systems and invariant manifolds, respectively, which are best read as a sequential block. Lyapunov functions are considered in chapter seven and bifurcations in nonautonomous systems in chapter eight. Generalizations to set-valued nonautonomous dynamical systems and nonautonomous semi-dynamical systems are treated in chapters nine and ten, while the effects of perturbations and discretization are discussed in chapter eleven. Up to here the state space is either \mathbb{R}^d or a general complete metric space, but in chapter twelve issues of explicit relevance to infinite-dimensional state spaces are considered. Chapter thirteen applies previous results to switching and affine control systems interpreted as nonautonomous dynamical systems, while chapter fourteen introduces readers to some of the differences arising in random dynamical systems due to their measure-theoretic rather than topological characteristics. The previous deterministic and random results are then applied to the synchronization of dissipative systems in chapter fifteen.

Finally, various background definitions and results needed within the text are given in the appendix.

Readers who are interested in the dynamical behavior of nonautonomous partial differential equations and evolution equations are advised to refer to the monographs of CARVALHO, LANGA & ROBINSON [35] and CHEPYZHOV & VISHIK [43] in conjunction with this book.

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