

# Contents

Preface	vii
Chapter 1. Autonomous dynamical systems	1
1. Introduction	1
2. Local asymptotic behavior	4
3. Global asymptotic behavior	12
4. Dependence on parameters	17
Chapter 2. Nonautonomous dynamical systems	23
1. Processes formulation	23
2. Skew product flow formulation	26
3. Entire solutions and invariant sets	31
Chapter 3. Attractors	37
1. Attractors of processes	38
2. Attractors of skew product flows	41
3. Existence of pullback attractors	44
4. Relationship between nonautonomous attractors	52
5. Upper semi-continuous dependence on parameters	55
6. Parametrically inflated pullback attractors	57
7. Pullback attractors with continuous fibers	60
8. Local attractors and repellers	62
Chapter 4. Morse decompositions	69
1. Attractor-repeller pairs	69
2. Morse decompositions	72
3. The one-dimensional case	75
Chapter 5. Linear systems	79
1. Exponential dichotomy	79
2. Dichotomy spectrum	82
3. Lyapunov spectrum	87
4. Morse decompositions	89
Chapter 6. Invariant manifolds	105
1. Global invariant manifolds	105
2. Local invariant manifolds	112
3. Hierarchies of invariant manifolds	114
4. Taylor approximation	116
5. Reduction principle	123

Chapter 7. Lyapunov functions	129
1. Lyapunov functions for solutions	129
2. Lyapunov functions for autonomous attractors	132
3. Lyapunov functions for pullback attractors	135
4. Lyapunov functions for Morse decompositions	143
Chapter 8. Bifurcations	147
1. Nonautonomous Bernoulli equations	147
2. One-dimensional bifurcation patterns	149
3. Higher-dimensional Bernoulli-like equations	157
4. Further developments	163
Chapter 9. Set-valued nonautonomous dynamical systems	169
1. Set-valued processes	170
2. Set-valued skew product flows	173
3. Backward extension of autonomous semi-dynamical systems	175
4. Proof of existence of nonautonomous invariant sets	178
Chapter 10. Nonautonomous semi-dynamical systems	185
1. Attractors of skew product semi-flows	185
2. The twisted horseshoe mapping	189
Chapter 11. Approximation and perturbation of attractors	191
1. Nonautonomous perturbations of an autonomous system	191
2. Numerical approximation of uniform attractors	193
3. Perturbation of the driving system	197
Chapter 12. Infinite-dimensional systems	205
1. Squeezing and flattening properties: the autonomous case	205
2. Pullback asymptotic compactness	207
Chapter 13. Switching and control systems	213
1. Switching systems	213
2. Affine control systems	222
Chapter 14. Random dynamical systems	227
1. Random attractors	228
2. The Ornstein–Uhlenbeck process	229
3. Random attractors for stochastic differential equations	231
Chapter 15. Synchronization	235
1. Deterministic nonautonomous systems	235
2. Synchronization of systems with additive noise	242
3. Synchronization of systems with linear noise	247
Appendix	251
Bibliography	253
Index	263