Preface

There is a vast literature devoted to the study of water waves ranging from coastal engineering preoccupations to a very theoretical mathematical analysis of the equations. Since all the scientific communities involved have their own approach and terminology, it is sometimes quite a challenge to check that they actually speak about the same things. The rationale of this book is to propose a simple and robust framework allowing one to address some important issues raised by the water waves equations. Of course, such a global approach is sometimes not compatible with sharpness; it has been a deliberate choice to sacrifice the latter when a choice was necessary (on minimal regularity assumptions, for instance). Hopefully, experts on the well-posedness of the water waves equations or on the mathematical properties of some asymptotic system, or on any other aspect, may, however, discover here other related subjects of interest and some open problems.

Since this book is addressed to various audiences, there has been an effort to make each chapter as thematically focused as possible. For instance, physical aspects are not often present in the chapters devoted to the well-posedness of the equations, and, vice versa, the chapter devoted to the derivation of the asymptotic models is rather oriented by physical considerations and requires only basic mathematical tools. More precisely,

- Chapter 1 is a general and basic introduction. It also includes a review of various approaches developed in recent years for the mathematical analysis of the water waves equations. Some extensions raising several new problems are also presented (such as moving bottoms and rough topography), and the physical assumptions made to derive the water waves equations are discussed.
- Chapters 2, 3, and 4 are devoted to the well-posedness of the water waves equations. They are addressed to mathematicians looking for a basic introduction to the Cauchy problem for water waves, as well as to researchers more familiar with these equations but not necessarily with their behavior in shallow water. Some aspects of the proof (e.g., the study of the Laplace equation, properties of the Dirichlet-Neumann operator) can also be of interest by themselves and we therefore gave sharper results than those actually needed to prove the well-posedness of the water waves equations.
- In Chapter 5, we derive many shallow water asymptotic models used in coastal oceanography. This chapter is addressed to oceanographers and people working on such models and those who want to know precisely what their range of validity is. This chapter uses only basic mathematical tools and should be readable independently of Chapters 2, 3, and 4. Note that the derivation of various models presented here (the so-called models with

improved frequency dispersion, for instance) is dictated by experimental considerations rather than mathematical ones (they are just likely to improve a multiplicative constant in some error estimates). Some numerical computations compared with experimental data show their relevance.

- Chapter 6 is a more mathematically oriented complement to Chapter 5. It gathers all the more technical steps necessary to provide a full justification of the models derived in Chapter 5 (i.e., error estimates between the approximation they furnish and the exact solution of the water waves equations constructed in Chapter 4). Some mathematical properties of these systems are also given here.
- Chapter 7 deals with approximations of water waves by *scalar* models. Many of these equations (KdV/BBM, KP, Camassa-Holm) have played a central role in mathematical physics in various contexts; they are derived and justified here for water waves.
- In Chapter 8 we address the deep water approximations and modulation equations. The latter are used to model the propagation of wave packets. These modulation equations appear in many physical situations and therefore play an important role in mathematical physics, for instance, the nonlinear Schrödinger, Davey-Stewartson, or Benney-Roskes equations. The full justification of these models still raises many open problems. We also derive several new variants of these equations, such as "full dispersion" versions that are expected to behave better than the standard models in some situations where the latter furnish a poor approximation.
- Chapter 9 analyzes the influence of surface tension. Since this physical effect is generally not relevant for applications to coastal oceanography, we did not include it in the previous chapters. However, capillary gravity waves raise many mathematical problems of interest and the results gathered here (e.g., well-posedness theory for capillary gravity waves, influence of the capillary effects on asymptotic models, etc.) may prove useful to those working in this area.
- In Appendix A, we provide additional results on the Dirichlet-Neumann operator (for instance, its analyticity properties, self adjointness, and invertibility). These results are not needed for our main purposes but are often needed in related topics. We tried to provide new proofs and/or improvements of the versions of these results that can be found in the literature.
- In Appendix B, we gather the product and commutator estimates used throughout this book so that the uninterested reader can use them as a black box.
- Finally, Appendix C presents in an extremely condensed form the models derived in Chapters 5, 7, and 9. It is aimed at helping the hurried reader to locate any asymptotic model among the dozens that are used to describe waves in several physical regimes (shallow water, modulation equations, etc.).

The material in this book can essentially be found in several papers by the author, such as [**12, 13, 223, 36, 107, 96, 225**]; therefore, this book owes a lot to the coauthors of these papers. However, most of the proofs of the results corresponding to these references have been changed; they have been simplified and/or improved using recent advances by different authors, in particular T. Iguchi [**181**], F. Rousset and N. Tzvetkov [**277**], and T. Alazard, N. Burq, and C. Zuily [**4, 6**].

New material has also been added on several theoretical aspects (in particular on the Dirichlet-Neumann operator); though these new results are not needed for the applications presented in this book, they are of general interest, and we hope that they will be useful to people working on this subject. We also derived several new models that are expected to improve existing versions. Most of them remain to be analyzed mathematically, and the quality of the approximation they furnish is yet to be investigated theoretically, numerically, and experimentally.

Many of the improvements and most of the new material gathered in this book were motivated by conversations with T. Alazard, J. Bona, P. Bonneton, A. Constantin, J.-C. Saut, and E. Wahlen, to whom I address my warmest thanks. I also want to express my gratitude to A. Castro, F. Chazel, V. Duchêne, and M. Tissier, who helped me with some illustrations and numerical simulations.

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