

Contents

Introduction	1
Chapter 1. The Rationality Problem	9
1. The classical heritage	10
2. Towards the modern birational geometry	19
3. Rationally connected varieties	28
Chapter 2. The Method of Maximal Singularities	37
1. Canonical adjunction	37
2. Exclusion of maximal singularities	48
3. Three-dimensional quartics	59
4. The connectedness principle	73
5. Complete intersections $V_{2,3} \subset \mathbb{P}^5$. I: Untwisting maximal singularities	84
6. Complete intersections $V_{2,3} \subset \mathbb{P}^5$. II: Excluding maximal singularities	93
Notes and references	110
Chapter 3. Hypertangent Divisors	115
1. Definitions and examples	115
2. Fano complete intersections	123
3. Regular Fano varieties	134
4. K -trivial structures	142
Notes and references	150
Chapter 4. Rationally Connected Fibre Spaces	153
1. Fano fibre spaces	153
2. The Sarkisov program	162
3. Birational rigidity of Fano fibre spaces	176
Notes and references	186
Chapter 5. Fano Fibre Spaces Over \mathbb{P}^1	189
1. Sufficient conditions of birational rigidity. I	190
2. Pencils of Fano hypersurfaces	202
3. Pencils of double hypersurfaces	213
4. Sufficient conditions of birational rigidity. II	234
5. Pencils of Fano complete intersections	241
Notes and references	252
Chapter 6. Del Pezzo Fibrations	253
1. Explicit constructions and a summary of known results	253

2. Infinitely near maximal singularities	261
3. Completing the proof for the pencils of cubic surfaces	273
Notes and references	283
Chapter 7. Fano Direct Products	285
1. Fano direct products	285
2. Inversion of adjunction	302
3. Fano varieties with elementary singularities	309
Notes and references	322
Chapter 8. Double Spaces of Index Two	323
1. Half-anticanonical pencils	323
2. Centres of codimension two and three	331
3. Counting multiplicities	336
4. Infinitely near singularities	345
5. Generic double spaces	350
Notes and references	356
Bibliography	359
Index	367