Index

ψ_v-regular sequence, *see also* regular sequence
w_v, *see also* distinguished place w_v

Abel map, **179**, 181, 187, **391**, 392, 409, 411, 417
adele ring, 193
affinoid
  admissible, 398
  affinoid domain, x, xviii
  Berkovich affinoid, 7, 123, 128
  domain, **398**, 401
  strict closed affinoid, 128–130
Akhiezer, Naum, 14
algebraic integer, xii, xiv, 27, 28, 30, 31, 33
  totally real, ix, xii
algebraically capacitable, xviii, xix, 1, 2, **4**, 5, 6, 20, 28, 39, **80**, 91, 106, 114, 117, 119, 120, 125, 129, 131, 159
  with respect to ζ, **80**
algorithm to compute nonarchimedean capacities, 22
analytically accessible, **89**, 104–107, 116
apportionment in House of Representatives, 160
Arakelov functions, **74**
Arakelov theory, xiii
arc, **334**, 385
  analytic, 378
  circular, 58
  smooth, 104, 105, 135
Argument Principle, 11, 261
Autissier, Pascal, xiii

Baker, Matthew, 5, 336
Balinski, Michael, 160
band
  $\mathrm{Band}_N(k)$, **214**, 215, 219, 221, 233, 234, **237**, 238, 239, 249, 250, 257, 259, 264, 269–272, 274, 280–284, 295, 296, 298, 301, 312, 313, 315–317
  coefficients patched by bands, 191, 214, 216, 221, 231, 234, 237
    simultaneously when $\mathrm{char}(K) = p > 0$, 231, 233–235
  high-order, 295
barrier, 334
Basic Patching Lemma, **287**, 291, 292, 309
basic well-distributed sequence, **98**, 171
basis
  L-rational, xv, 61, 64, **65**, 66–69, 191, 197, 209, 211–213, 219, 224, 230–232, 234, 243, 244, 247–249, 251, 257, 262, 263, 269, 271, 275, 280, 282, 299–301, 326–329
  L^{sep}-rational, 64, 65, **66**, 67–69, 197, 198, 224, 230, 231, 234, 235, 237, 238, 241, 243, 247, 248, 269, 280, 328
  scaled L-rational, **242**, 243, 246
Berkovich
  adelic neighborhood, 6, 129
  adelic set, 6, 7, 128, 129
  analytic space, 5, 120, 336
  analytification, 120, 125
  closure, 121, 125, 128
  compact set, 130
  curve, xii, xix, 120, 122
  Green's function, 120, 125
  neighborhood, 7, 128
  open set, 7, 130
  quasi-neighborhood, 6, 7, 128–130
  strict closed affinoid, 7, 128, 130
  topology, 6
Berkovich, Vladimir, 5, 120
Bertrandias, Françoise, xii
binomial theorem, 251
block, Galois
  $\mathrm{Block}(i, j)$, **214**, 220, **237**, 238
  coefficients patched block by block, 214, 219, 220
boundary, 111
  equilibrium distribution supported on, 37
  exceptional set contained in, 81
  of filled Julia set, 19
  piecewise smooth, xiii, 103, 105, 133, 135, 202, 249, 257, 261
Brouwer Fixed Point theorem, xv, xxi, **154**

$\mathbb{C}$-simple set, **103**, 105, 133, 134, 200, 202
$\mathcal{C}(\widetilde{K}_v^{\mathrm{sep}})$ is dense in $\mathcal{C}_v(\mathbb{C}_v)$, 64
canonical distance
  $[z,w]_{\mathfrak{X},\vec{s}}$, xx, xxi, **76**, 73–77, **137**, **163**, 171, 176, 177, 331, 351
    normalization of, 163
    potential theory for, xxi, 137, **331**
  $[z,w]_{\infty}$, 164
  $[z,w]_{\zeta}$, xv–xvii, 22, 23, 44, 61, **73**, 74–78, 117, 119, 125, 126, 137, 138, 351, 403
    normalization of, **xvi**, 44, **73**, 331, 342, 345, 351
  $[z,w]_{\zeta}^{\mathrm{an}}$, **126**
  archimedean
    comparable with absolute value, 333
    deviation from absolute value, xxi
  nonarchimedean
    comparable with chordal distance, 333
    constant on disjoint balls, 22, 165, 177
    intersection theory formula, **44**, 51
  change of pole formula, 75
  constructed
    directly, xvii, 74
    in good reduction case, 75
    using Arakelov functions, 74
    using Néron's pairing, 74
  continuity of, **73**
  determines 'shortness', xxi, 138
  factorization property, xx, **73**, 176, 396, 403, 405
  Galois equivariance, **73**
  symmetry of, **73**
Cantor's Lemma, **48**
Cantor, David, ix, x, xii, xv, xix, 20, 28, 30, 31, 48, 196
capacity, xx, 6, 9, 10, 12–14, 17, 20, 21, 23, 25, 26, 28, 41, 45, 50, 51, **78**, 80, 144, 178, 333, 352, 353
  Cantor capacity, xi, xii, xiv, xv, **xvi**, xvii, **xviii**, xix, 1, 2, 4, 31, 39, 91, 93, 192
    inner Cantor capacity, **1**, **2**, 4, 61, **91**, **93**, 94, 192
    functoriality properties of, 94
  Choquet capacity, **122**
  inner capacity, **xvii**, xviii, 2, 3, 5, 22, 23, **79**, 80, 81, 86–88, 115
  logarithmic capacity, xvi, 2, 331, 353
  outer capacity, **79**, **80**
  scaling property of, 25, 34
  Thuillier capacity, 121
  $(\mathfrak{X},\vec{s})$, xxi, xxiii, **79**, 163, **331**, **332**, 352
    weighted $(\mathfrak{X},\vec{s})$, 352, **353**, 366, 372
  sets with capacity 0, **78**, 79, 81–83, 85, 86, 89, 90, 116, 120, 126, 163, 332–335, 338, 340, 344, 347, 348, 355–357, 360, 362, 365, 367, 369, 373, 377
  sets with positive capacity, 20, **78**, 79, 81, 83, 85, 86, 88–90, 92, 125, 126, 133–135, 137, 144, 159, 162, 163, 333, 334, 336, 339, 340, 343, 344, 346–348, 354, 355, 367, 369–371, 377
capacity theory, 27, 31, 61
carrier, **355**
Cauchy's theorem, 15
Chebyshev constant, xxiii, 331, 352
  $(\mathfrak{X},\vec{s})$, xxiii, 331, 332, 352
  restricted, 332, 356
  weighted, 352, **356**, 357, 361, 369, 370, 375, 385
  weighted $(\mathfrak{X},\vec{s})$, 352, 366
Chebyshev measures, 352, 376, 378
  converge weakly to $\mu_{\mathfrak{X},\vec{s}}$, **376**
Chebyshev points, 386
Chebyshev polynomial, xv, xxi–xxiii, 29, 33, 35, 134, 138, 211, 215, **258**, 259, 260, 265, 351, 352, 378–380
  mapping properties, **258**
  restricted, 356
  weighted, 352
  weighted $(\mathfrak{X},\vec{s})$, 382
Chebyshev pseudopolynomial, 352
  restricted, 351
  weighted, 352, 357, 375, 376
  weighted $(\mathfrak{X},\vec{s})$, 378, 379, 382, 385
Choquet capacity, **122**
chordal distance, x, **69, 70**
Clark, Pete L., xxiv, 57
closure of $\mathcal{C}_v(\mathbb{C})$ interior, 103, 105, 111, 133, 202, 249, 257, 384
coefficients $A_{v,ij}$, xiv, xv, 191, 199, 203, 211–217, 221, 233, 234, 249, 250, 252, 257, 259, 262, 269, 272, 280, 283, 289–291, 296, 298, 329
  L^{sep}-rational, 234
  growth rate, 234
  leading, xv, xvi, xxii, 161, 162, 170, 187, 199, 200, 202, 205–207, 209, 211–215, 217–221, 224, 225, 227–231, 233, 235–237, 249, 250, 252, 253, 257–259, 264, 265, 269–272, 274–276, 280–284, 295–301, 303, 304, 308, 309, 312, 313, 315, 317, 325, 326, 328
  high-order, xiv, xvi, xxii, 191, 211, 213, 219, 220, 231, 238, 251, 261–263, 272–275, 284, 295, 300, 305–309, 311
  middle, xxii, 191, 221, 238, 253, 254, 264, 272, 277, 284, 314, 315
  low-order, xxii, 191, 222, 240, 254, 266, 272, 277, 284
  patching, 221, 296
  restoring, 305, 306, 309, 327
  target, 213, 238
coherent approximating functions $\phi_v(z)$, xv, 199, 200, 203, **204**, 211, 213, 214, 226, **227, 228**, 231, 249, 257, 269, 279
  construction when $\mathrm{char}(K) = 0$, 204

choice of the initial approximating functions, 206
preliminary choice, 207
adjusting absolute values of leading coefficients, 207
making the leading coefficients S-subunits, 209
construction when $\mathrm{char}(K) = p > 0$, 227
choice of the coherent approximating functions, 228
choice of the initial approximating functions, 228
making the leading coefficients S-subunits, 229
compatible with $\mathfrak{X}$, x, xi, xviii, 1–5, 39, 40, 63, 91, 94, 103, 104, 110, 115, 116, 119, 128, 191, 193, 203
Berkovich set compatible with $\mathfrak{X}$, 6, 7, 127, 128
compensating functions $\vartheta^{(k)}_{v,ij}(z)$, **212**, 215, 216, 233, 234, 250–252, 254, 259, 263–266, 272, 277, 281, 284, 290, 296, 298, 299, 304, 312, 315, 317
are K_v-symmetric, 215, 216, 233, 259, 263–265, 270, 274, 281, 289, 298, 313, 315, 317
bounds for, 254, 265, 274, 312
construction of, 251–253, 264, 270, 272, 274, 281, 296, 298, 304, 312, 315, 316
more complicated than basis, 212
poles and leading coefficients of, 212, 215, 219, 221, 233, 250, 252, 253, 259, 265, 270, 274, 281, 289, 296, 298, 304, 312, 315
confinement argument, xvi, xxii, xxiii, 211, 214, 231, 249, 257, 269, 279
Courant, Richard, 11, 18, 19
Cramer's Rule, 49, 166
Cremona, John, 51–53
crossratio, 389, 394
cusps, 57, 58, 60

δ_n-coset, **306**, 309–311, 313
degree-raising, 98, 199, **211**, 217
Deligne-Rapoport model, **57**, 60
Determinant Criterion
for negative definiteness, 32, 37
differential
holomorphic, 140, 142, 143, 145, 152
meromorphic, 141–143, 145
of the third kind, 142
real, 140, 155
distinguished balls, 187, 321
distinguished boundary, **273**
distinguished place w_v, **62**, 207, 213, 215, 218–222, 232, 234, 236–241, 248–250, 257, 259, 264, 269–272, 274–277, 279, 282, 283, 289, 290, 295, 297, 298, 301, 302, 307, 308, 312, 313, 315–317, 328
Dominated Convergence theorem, 83, 91

elliptic curve, xx, 9, 40, 52, 53
embedding $\iota_v : \widetilde{K} \hookrightarrow \mathbb{C}_v$, **62**
energy integral, **xvii**, **78**, 90, 125
$(\mathfrak{X}, \vec{s})$, 335, 339, 343, 371
classical, **xiii**
equilibrium distribution, xiv, xv, xvii, 81, 125, 335–339, 342, 343, 347, 348, 356, 361, 367, 376
$(\mathfrak{X}, \vec{s})$, xxi, xxiii, 331, **333**–**336**, 338–345, 347, 348, 367, 371, 373, 376, 378, 382, 384, 387
classical, **xiii**
determining, **335**, 338, 341
weighted $(\mathfrak{X}, \vec{s})$, 372, 374, 375
equilibrium potential, 335, 336, 338, 342, 346–348, 353
$(\mathfrak{X}, \vec{s})$, **331**, **333**, 334–336, 338, 342–344, 346, 348, 349, 352, 373, 376, 377, 382, 384–386
determining, 342, 344
is lower semi-continuous, 377
is superharmonic, 385
takes constant value a.e. on E_v, 333–335, 338, 343, 344, **355**, 373, 377
escape velocity, 19
examples
archimedean
for the disc, 9
for one segment, 10
for two segments, 10
for three segments, 14
for multiple segments, 14
for the real projective line, 17
for a disc with arms, 18
for two concentric circles, 18
for Julia sets, 19
for the Mandelbrot set, 20
nonarchimedean
for a closed disc, 20
for an open disc, 21
for a punctured disc, 21
for the ring of integers $\mathcal{O}_w$, 21
for the group of units $\mathcal{O}_w^\times$, 23
for an annulus K_v, 24
with nonrational Robin constant, 25
global examples on $\mathbb{P}^1$
for the Mandelbrot set, 27
for the segment $[a, b]$, 28
of Moret-Bailly type, 29
contrived, 29
with overlapping sets, 30
an example of Cantor, continued, 30
Robinson's unit theorem, 31
for units in two segments, 32

S-unit analogue, 33
example with $E_\infty \cap \mathfrak{X} \neq \phi$, 35
regarding Ih's conjecture, 36
with units near circles, 38
concerning separability hypotheses
for function fields , 38
for elliptic curves
archimedean pullback sets, 40
theorem using Néron-Kodaira classification of special fibres, 42
nonarch Weierstrass equations, 50
global, Cremona 50($A1$), 51
global, non-minimal Weierstrass equation, 52
global, Cremona 48($A3$), 52
global, Cremona 360($E4$), 53
for Fermat curves, 53
for the modular curve $X_0(p)$, 57
exceptional set, 81, 82, 333, 334, 344

Falliero, Thérèse, 12–14
Fekete
measure, 367, **368**, 369, 371, 373–375
points, **361**, 362, 382, 383, 386
(n, N), 362
Fekete's theorem, xii, 4, 5, 30, 39, 80, 120
Fekete, Michael, ix, xi, xii, 212, 260, 411
Fekete-Szegö theorem, xi–xiv, 1, 4, 5, 37, 80
Fekete-Szegö theorem with Local Rationality, xiii, xiv, xvi, xix–xxi, 1, 9, 30–32, 34–38, 52, 53, 56, 57, 60, 63, 74–77, 80, 93, 94, 134, 160
need for separability hypotheses in, xi, 38
for K_v-simple sets, **103**
producing points in $\mathbb{E}$, **xi** , **104**
for Incomplete Skolem Problems, **108**
for quasi-neighborhoods, xix, 2, **3**, **110**
Strong form, **3**, **115**
and Ramification Side Conditions, **4**, **116**
for algebraically capacitable sets, **5**, **119**
Berkovich, 5, **7**, **127**
for Berkovich quasi-neighborhoods, **7**, **128**
Fekete-Szegö theorem with splitting conditions, ix, xi–xiii
Fermat curve, xx, 9, 53, 56
McCallums's regular model for, 55
diagram of special fiber, 55
filled ellipse, **258**
finite K_v-primitive cover, x, 104
First Moving Lemma, **307**, **318**
formal group, xxiv, 42, 179, 180, 408, 410, 414, 415, 418–421
freedom B_v in patching, **212**, 213, 250, **252**, **262**
Frobenius' Theorem, **xviii**
Frostman's Theorem, 333, 355
Fubini-Study metric, x, 69, **70**
Fubini-Tonelli theorem, 83, 91, 335, 376, 377, 385
Fundamental Theorem of Calculus, 15, 16

Gauss norm, 412
global patching when $\mathrm{char}(K) = 0$
outline of Stages 1 and 2, 199
Stage 1: Choices of sets and parameters
summary of the Initial Approximation theorems, 200
the K_v-simple decompositions, 202
the open sets U_v, 202
the sets $\widetilde{E}_v$, 203
the local parameters η_v, $\mathcal{R}_v$, h_v, r_v, and R_v, 203
the δ_v for $v \in \widehat{S}_{K,\infty}$, 204
the probability vector $\vec{s}$, 204
Stage 2: The Approximating Functions
Coherent approximation theorem, 204
the choice of N, 205
Initial approximating functions, 206
preliminary choice of the Coherent approximating functions, 207
adjusting the leading coefficients, 207
Coherent approximating functions, 209
Stage 3: The global construction
overview, 211
details, 213
the order $\prec_N$, 213
summary of Local patching theorems, 214
the choices of $\overline{k}$ and B_v, 216
the choice of n, 217
patching leading coefficients, 218
patching high-order coefficients, 219
patching middle coefficients, 221
patching low-order coefficients, 222
conclusion of the argument, 222
constructing points in Theorem 4.2, 223
global patching when $\mathrm{char}(K) = p > 0$
Stage 1: Choices of sets and parameters
the place v_0, 224
summary of the Initial approximation theorems, 224
the K_v-simple decompositions, 225
the probability vector $\vec{s}$, 225
the sets $\widetilde{E}_v$, 225
the parameters η_v, h_v, r_v, and R_v, 226
Stage 2: The Approximating functions
Coherent Approximation theorem, 227
the choice of N, 228
choice of the Initial approximating functions, 228
choice of the Coherent approximating functions, 228
adjusting the leading coefficients, 229
Stage 3: The global construction

overview, 231
summary of Local patching theorems, 231
comparison with $\mathrm{char}(K) = 0$, 233
the patching by Blocks theorem, 235
the choice of $\overline{k}$, 236
the choice of n, 236
the order $\prec_N$, 237
patching high-order coefficients, 238
patching leading coefficients, 237
patching low-order coefficients, 240
patching middle coefficients, 238
conclusion of the argument, 241
constructing points in Theorem 4.2, 242
good reduction, x, 6, 51–53, 56, 60, 62, 63, 71, 75, 92, 160, 192, 213, 223, 224, 242
Grauert's theorem, xxiii, 390, 392
Green's function, **xiii**, xiv, xvii, xx, 2, 9, **81**–**83**, 93, 117, 125, 192, 193, 208, 224–226, 335–339, 340, 346–349, 386
$(\mathfrak{X}, \vec{s})$, xxi, xxiii, 137, 163, 331, **335**–**336**, 340, 386, 387
archimedean, 135, 136
characterization of, **9**, 10, 14, 18, 19
guessing, 9
properties of, 145, 150
nonarchimedean, 159–161, 200–203, 208
$(\mathfrak{X}, \vec{s})$, 163, 168
takes on rational values, **88**, **164**, **346**
Berkovich, 5, 6, 120, 125
characterization of, 123, 124, 126
compatible with classical, **125**, 126
monotonic, 127, 129, 131
properties, **120**, 128, 130
computing nonarchimedean, 20, 22, 166
continuous on boundary, 147
examples, xx
archimedean, 9–15, 18–20, 32, 35
nonarchimedean, 18, 20–24, 32, 117
elliptic curve, 41, 42
Fermat curve, 56
modular curve, 58, 59
identifying, **89**
lower, **81**, 89, 192
monotonic, xvii, 20, **82**, 85, 107, 108, 111–115, 119, 135, 146, 150, 347
of a compact set, **xiv**, **81**, 87–89, 114, 133, 192
properties of, xvii, **81**, **88**, 94, 104–108, 111, 113, 115, 116, 133, 144
pullback formula for, 11, **20**, 21, 34, 41, 54, 58, **87**
Thuillier, 5, 120, 121, 123, 126
upper, 1, 9, 61, **81**, **85, 86**, 88, 89, 91, 94, 117, 119, 133, 192, 347
upper $(\mathfrak{X}, \vec{s})$, 334, 335
Green's matrix
global, xviii, xix, 2, 5, 31, 32, 34, 37, 38, 56, 60, 91, 104, 111, 119, 120, 128, 130, 134, 192, 199, 203, 225, 226
global Berkovich, 6, 130
upper global, **2**
local, 32, 34, 37, 38, 54–60, 92, 192
local Berkovich, 6
upper local, xviii, **2**
negative definite, xviii, 2, 5, 32, 34, 37, 38, 93, 117, 119, 120, 192, 193, 203, 204, 225, 226
Green, Barry, xii
Grothendieck, Alexander, 39
group chunk, 408

Haar measure, xvi, 22, 61, 163–165, 342, 343, 345
Harnack's Principle, 88
Berkovich Harnack's Principle, 123–125
Hilbert scheme, 391, 393
Hilbert, David, 11, 18, 19
homogeneous coordinates
choice of, **62**

idele group, 193
Incomplete Skolem Problems, xi, 108
independent variability
of logarithmic leading coefficients, xv, 94, **134, 135**, 200, **204**
indices, **306**, 309, 310, 319, 322
safe, **306**, 308, 313
unpatched, **306**, 313
consecutive, 314
initial approximating functions $f_v(z)$, xiv, xv, xxi–xxiii, 161, 191, 199, 200, 203–206, 213, 217, 226–228
archimedean, **133**, 134, 339
nonarchimedean, **159**, 160, 161, 341, 407
construction when $K_v \cong \mathbb{C}$, **134**, 135
construction when $K_v \cong \mathbb{R}$
outline, 136–141
independence of differentials, 141–144
Step 0: the case $E_v \cap \mathcal{C}_v(\mathbb{R}) = \phi$, 144
Step 1: reduction to short intervals, 144–145
Step 2: the choice of $t_1, \ldots, t_d$, 145–147
Step 3: the choice of r, 147–150
Step 4: the construction of $\widetilde{E}_v$, 150–151
Step 5: study of the total change map, 151–154
Step 6: the choice δ_v, 154–155
Step 7: achieving principality, 155–156
Step 8: the choice of N_v, 156–157
construction for nonarchimedean
RL-domains, 160, **161**

construction for nonarchimedean K_v-simple sets, 160, **161**, 162–189
Step 0: reduction to the case of a single ball, 162–171
Step 1: construction of generalized Stirling polynomials, 171–174
Step 2: reduction to finding a principal divisor, 175–177
Step 3: the proof when $g(\mathcal{C}_v) = 0$, 177–178
Step 4: the local action theorem, 179–180
Step 5: the proof when $g(\mathcal{C}_v) > 0$, 181–186
consequences of the construction, 186–189
Initial Approximation theorems
when $K_v \cong \mathbb{C}$, **134**
when $K_v \cong \mathbb{R}$, **135**
for nonarchimedean RL-domains, **161**
for nonarchimedean K_v-simple sets, **161**
summary of the Initial Approximation theorems
when $\mathrm{char}(K_v) = 0$, 200
when $\mathrm{char}(K_v) = p > 0$, 224
Initial Patching functions, *see also* patching functions, initial $G_v^{(0)}(z)$
inner capacity, **79**
Institute Henri Poincaré, xxiv
Intermediate Value theorem, 154
Intersection Theory formula
for the canonical distance, **44**, 51
irreducible matrix, **120**
isometric parametrization, 45, 71, 72, 76, 89, 107, 114, 163, 164, 169, 176, 177, 179–184, 244
isometrically parametrizable ball, 4, **71**, 76, 77, 82, 88–90, 97, 103, 107, 112, 114, 117, 118, 121, 159, 160, 162, 164–166, 169, 171, 174–177, 180, 181, 201, 205, 224, 227, 235, 243, 246, 347

Jacobi identity, 19
Jacobi Inversion problem, 15
Jacobian Construction Principle, 74
Jacobian elliptic function, 12
Jacobian variety, xvi, xxiii, 74, 139, 155, 389, 391, 407–411, 414, 418–421
structure of $\mathrm{Jac}(\mathcal{C}_v)(\mathbb{R})$, 140
Jordan curve, x, 7, 104, 127, 128
Joukowski map, 10, 17
Julia set, 19, 20, 28
filled, 19, 20

K-symmetric, **63**
index set, 213
matrix, 225, 226
probability vector, 94, 199, 204, 208, 226
set of numbers, 194, 197, 205, 239, 240
set of points, 159
system of subunits, 210
system of units, 210, 217, 218, 228, 230, 236, 237
vector, 206, 209, 210, 213, 230
K_v-symmetric, **63**, 207, 249, 262–264
divisor, 139, 157
function, 263, 264
probability vector, 133, 136, 138, 141, 145, 156, 159, 161, 162, 167, 170, 188, 201, 202, 224, 225, 249, 257, 258, 269, 271, 279, 280, 282, 351, 382, 384, 387
quasi-neighborhood, **2**
set of functions, 68, 199, 202, 215, 216, 218, 230, 232, 233, 237, 238, 248, 262, 270, 272, 274, 276, 281, 283, 289, 296, 298, 302, 307, 313, 315, 317, 320
set of numbers, 134, 202, 210, 212, 215, 218, 219, 230, 232, 234, 237, 240, 250, 259, 262–264, 266, 270, 271, 275, 277, 281–283, 290, 295–298, 301, 307, 312, 315–319, 328
set of points, 307, 310, 319, 320
set of roots, 298
set of vectors, 248
system of units, 297, 301
vector, 136, 138, 144, 156, 157, 201, 236, 248, 382, 384, 387
K_v-primitive, **x**, 104
K_v-simple
$\mathbb{C}$-simple, **103**, 133, 202
$\mathbb{R}$-simple, **103**, 133, 202
decomposition, **103**, 162, 163, 165, 167–169, 186–188, 201–206, 214, 225, 227, 228, 231, 279, 280, 282, 307, 309, 318–321, 327
decomposition compatible with another decomposition, **161**, 162, 169, 171, 186, 189, 202, 205, 206, 225, 227, 228, 279, 280, 282, 318, 319
set, xxi, **103**, 104, 108, 133, 136, 138, 144–146, 150, 157–162, 164–167, 169, 171, 186, 188, 191, 201, 202, 205, 224, 225, 227, 233, 241, 242, 258, 279, 280, 282, 307, 309, 318, 319, 327
set compatible with another set, **161**, 167, 186, 201, 224, 225
K_v is separable over K, 39
Kazez, William, xxiv, 383
Kleiman, Steven, 391
Kodaira classification of elliptic curves, 42

$\log(x)$
means the natural logarithm, **xiii**, **61**
$\log_v(x)$
definition of, **xvi**, **61**
L-rational basis, 61, 64–69

definition of, **65, 66**
uniform transition coefficients, **67**
transition matrix is block diagonal, xxi, **67**, 328
growth of expansion coefficients, **68**
good reduction almost everywhere, **68**
rationality of expansion coefficients, **68**
multiplicatively finitely generated, 69, 192, 243
uniform growth bounds, **69**
L^{sep}-rational basis, 64–69
definition of, **65, 66**
Lang, Serge, 65
lattice, 195
Laurent expansion, 305, 325, 326
Lipschitz continuity
of the Abel map, 321, **411**, **421**
local action of the Jacobian, xvi, xxiii, **179**, 181–184, 187, 319, 321–323, **410**, 407–421
local patching constructions:
local patching for $\mathbb{C}$-simple sets
Phase 1: high-order coefficients, 251–253
Phase 2: middle coefficients, 253–254
Phase 3: low-order coefficients, 254
local patching for $\mathbb{R}$-simple sets
Phase 1: high-order coefficients, 261–264
Phase 2: middle coefficients, 264–266
Phase 3: low-order coefficients, 266
local patching for nonarchimedean RL-domains
Phase 1: high-order coefficients
when $\text{char}(K_v) = 0$, 273–274
when $\text{char}(K_v) = p > 0$, 274–276
Phase 2: middle coefficients, 277
Phase 3: low-order coefficients, 277
local patching for nonarchimedean K_v-simple sets
Phase 1: leading and high-order coefficients
when $\text{char}(K_v) = 0$, 295–299
when $\text{char}(K_v) = p > 0$, 299–303
Phase 2: carry on, 303–305
Phase 3: move roots apart, 305–311
Phase 4: using the long safe sequence, 311–313
Phase 5: patch unpatched indices, 313–316
Phase 6: complete the patching, 316–318
logarithm
$\log(x)$ means $\ln(x)$, **xiii**, **61**
definition of $\log_v(x)$, **xvi**, **61**
logarithmic leading coefficients, xxi, xxii, **134**, 136, 141, 147, 151, 159, 160, 202, 204, 386
independent variability of archimedean, xv, 94, **134, 135**, **155**, 200, **204**
of $Q_{\vec{n}}(z)$, 386
logarithmically separated, **311**, 314, 316
long safe sequence, **306**, 311, **312**
Lorenzini, Dino, xxiv, 54
lower triangular matrix, **245**

magnification argument, xvi, xxii, **213**, **216**, 231, **250–252**, 259, **261–263**
Mandelbrot set, 20, 27
Maple computations, 27, 36, 38, 46, 48, 52, 53, 57
Maria's Theorem, 333
mass bounds, xxiii, **146, 147**, **339**
matrix
irreducible, 120
negative definite, 110
Matsusaka, Teruhisa, 391, 408
Maximum principle
for harmonic functions, 16, 333, 375, 384
strong form, 336, 377
for holomorphic functions, 253, 260
for superharmonic functions, 385
nonarchimedean, 396
for RL-domains, 273, 403, 404
for power series, 72, 308, 325, 412, 414
from Rigid analysis, 396, 401
McCallum, William, xx, 55, 57
Mean Value theorem, 379
Mean Value theorem for integrals, 152
Milne, James, 391, 408
minimax property, xii, 6, 31, 60, 94, 226
modular curve, xx, 9
$X_0(p)$, 57, **60**
Deligne-Rapoport model, xx, **60**
Modular Equation, **57**
modular function $j(z)$, 57
Monotone Convergence theorem, 84, 91
Moret-Bailly, Laurent, xi, xii, xix, 29
move roots apart, 284, 305
move-prepared, **186**, 187, 188, 202, 205, 206, 225, 227, 228, 279, 280, 282, 305, 318, 321
multinomial theorem, xiv, xvi, 264
multivalued holomorphic function, 12
Mumford, David, 11

n astronomically larger than N, 211
Néron model, **408**
of elliptic curve, xx, 41, 50
of Jacobian, 179
Néron's local height pairing, **74**
National Science Foundation, **xxiv**
disclaimer, xxiv
Newton Polygon, xvi, xxiii, **94–97**, 244, 287–289, 291, 292, 325
nonpolar set, **6**, 120–123, 125, 126, 128
numerical
computations, 9, 13
criteria, xx
examples, xx, 14, 26

order $\prec_N$, **213**, **214**, 215, 219, 231, 234, **237**, 238, 249–252, 254, 257, 259, 263, 264, 269, 270, 274, 280, 281, 295, 296, 298, 312, 315, 317
ordinary point, **58**, 59, 60
outer capacity, **79**

PL-domain, 89
PL_ζ-domain, **79**
Park, Daeshik, xi, xiii, 38
partial self-similarity, 25
patched roots, 305
patching constructions
  origins of, xiii
  global, xv, xxi, xxii, 64, 98, 191, 192, 194, 199, **211**, 212, **213**, 214, 217, 219, **231**, **236**, 249, 265, 275, 290
    when $\mathrm{char}(K) = 0$, 199–223
    when $\mathrm{char}(K) = p > 0$, 223–242
  comparison of $\mathrm{char}(K) = 0$ and $\mathrm{char}(K) = p > 0$, 233, 234
  tension between local and global, 212
  conclusion of global, 222, 223, 241, 242
  see also global patching constructions
  local, xvi, xxii, xxiii, 38, 186, 191, **211**–**214**, 216–218, 221, **231**, 232, 233, 236–238, 249, 250
  freedom B_v in patching, 250, 259, 262
    for the case when $K_v \cong \mathbb{C}$, 249–255
    for the case when $K_v \cong \mathbb{R}$, 257–267
    for nonarchimedean RL-domains, 269–277
    when $\mathrm{char}(K_v) = 0$, 269
    when $\mathrm{char}(K_v) = p > 0$, 270
    differences when $\mathrm{char}(K) = 0$ and $\mathrm{char}(K) = p$, 272
  for nonarchimedean K_v-simple sets, 279–329
    differences when $\mathrm{char}(K) = 0$ and $\mathrm{char}(K) = p$, 284
    the Basic Patching lemmas, 284, 287
    the Refined Patching lemma, 290
    proofs of the three Moving lemmas, 318–329
  see also local patching constructions
patching functions, Initial $G_v^{(0)}(z)$, xv, xvi, 37, 98, **217**, **236**, 250, 272, 275, 276, 281, 284, 296, 301, 302, 308
  are K_v-rational, 276, 296
  construction of, xv, 199, 211, 215, 217, 231, 237, 250, 251, 259, 260, 263, 270, 271, 274, 275, 282, 284, 294, 300
  expansion of, 232, 237, 262, 271, 275, 282, 299, 300
  for archimedean sets E_v
    patched by magnification, 251, 252, 262, 263
  for nonarchimedean K_v-simple sets
    are highly factorized, 284, 289, 296
    roots are distinct, 282, 295
  leading coefficients of, 207, 217, 218, 231, 236, 252, 270, 271, 275, 281–283, 300
  making the leading coefficients S-units, 217, 218, 237
  mapping properties of, 260–262, 275, 276
  roots confined to E_v, 231, 236, 261, 282
  when $\mathrm{char}(K) = p > 0$, 282, 299–301
patching functions $G_v^{(k)}(z)$ for $1 \le k \le n$, 215, 216, 227, 232, 234, 237, 238, 241, 251, 265–267, 274, 283, 305, 311, 318
  leading coefficients of, 232, 252, 270, 272, 281, 283, 297, 298, 301, 312
  expansion of, xv, xvi , 211, 231, 233, 239, 240, 274, 289, 298, 302, 326–328
  factorization of, 289, 297–299, 303–305, 312, 313, 315, 316
  mapping properties of, 265–267, 318
  are K_v-rational, 213, 231, 234, 240, 264, 274, 276, 277, 290, 296–298, 302, 305, 307, 313, 315, 317, 318, 328
  viewed simultaneously over K_v and L_w, 217, 218, 222, 238, 240
  modified by patching, xv, 199, 211, 212, 215–217, 219–222, 232–235, 237, 238, 240, 241, 250, 251, 253, 254, 259, 264–267, 270–272, 274, 276, 277, 281, 283, 284, 290, 295–298, 301, 303–305, 307–309, 311–313, 315–318, 324, 329
  for archimedean sets E_v
    oscillate on real components of E_v, 259, 264, 267
    patched by magnification, 263, 264
  for nonarchimedean K_v-simple sets
    movement of roots, 296, 297, 299, 302, 303, 311, 313, 314, 316
    roots are distinct, 233, 281, 284, 295
    roots are separated, 216, 284, 308, 311, 314, 316–318
  roots confined to E_v, xv, 212, 216, 218, 223, 233, 234, 238, 240, 242, 255, 259, 260, 265–267, 270, 272, 274, 276, 277, 284, 290, 302, 317
  $G_v^{(n)}(z) = G^{(n)}(z)$ is independent of v, xv, 199, 213, 222, 241
patching parameters, 192, 199, **202**, 203, 204, 206, 213, 214, 216, 218, 224, 227, 231, **236**, 249, 257, 269, 280
  choice when $\mathrm{char}(K) = 0$, **203**, **205**
  choice when $\mathrm{char}(K) = p > 0$, **224**–**228**
patching ranges, xiii–xv, **212**
  leading coefficients, xv, xvi, xxii, **218**, 236, 237
  high-order coefficients, xvi, 219, 231, 238, 251, 261, 273
    for RL-domains when $\mathrm{char}(K_v) = 0$, 273

for RL-domains when $\mathrm{char}(K_v) = p > 0$, 274
middle coefficients, 221, 238, 253, 264, 277
low-order coefficients, 222, 240, 254, 266, 277
patching theorems
for the case when $K_v \cong \mathbb{C}$, 249
for the case when $K_v \cong \mathbb{R}$, 258
for nonarchimedean RL-domains
when $\mathrm{char}(K_v) = 0$, 269
when $\mathrm{char}(K_v) = p > 0$, 271
for nonarchimedean K_v-simple sets
when $\mathrm{char}(K_v) = 0$, 280
summary of the local patching theorems
when $\mathrm{char}(K_v) = 0$, 214
when $\mathrm{char}(K_v) = p > 0$, 231
global patching theorems
when $\mathrm{char}(K) = 0$, **211**
when $\mathrm{char}(K) = p > 0$, **231**
period lattice, 11, 140, 155
Picard group , relative, 409
Picard scheme, xxiii, 179, 391, 394, 409
Pigeon-hole Principle, 72, 306, 312
Poincaré sheaf, 391, 392, 394
polar set, 126
Pólya-Carlson theorem, xii
Pop, Florian, xii
potential function, 22, **81**, 82, 83, 90, 172, 334, 335, 337, 342, 354–356, 375–377
$(\mathfrak{X}, \vec{s})$, xxiii, 137, 163, 331, 340, 342, 346
is lower semi-continuous, 355, 377
is superharmonic, 355, 377
of $\mathcal{O}_w$, 22, 164
properties of, 354
takes constant value a.e. on E_v, 125
potential theoretic separation, 339
potential theory, 79, 351, 352
$(\mathfrak{X}, \vec{s})$, xxi, xxiii, 76, 137, **331**
arithmetic, xvii, 74
classical, xxi, xxiii, 352
on Berkovich curves, 5, 120
weighted, 352, 371
Prestel, Alexander, xiii
Primitive Element theorem, 64
principal homogeneous space, xxiii, 160, 179, 180, 407, 409, 410, 419, 420
pro-p-group, 408
pseudoalgebraically closed field, 394
pseudopolynomial, **77**, 78, **137**, 138, 139, 147, 151, 352
$(\mathfrak{X}, \vec{s})$, xxi, **77**, 137, 138, 147, 148, 156, 175, 332, 336, **351**, 382–384, 387
Chebyshev, 351, 352
restricted, 351
special, **139**, 151, 154, 156
weighted $(\mathfrak{X}, \vec{s})$, 357, 369, 378
weighted Chebyshev, 352, 357, 375, 376, 379, 380, 382, 385
pure imaginary periods, 14–16, 141, 143

q_v , definition of, **xvi**, **61**
quasi-diagonal element, 197
quasi-interior, **133**, 135, 144, 257
quasi-neighborhood, xii, xix, **xxii**, 1, **2**, 3, 5, 110–112, 116, 119, 120
Berkovich, 6, 7, 128–130
separable, **3**, 5, 110, 116, 119, 131

RL-component, **160**, 161
RL-domain, x, **x**, xxi, xxii, 3–6, 33, 37, 80, 88, 89, 106, 107, 129–131, 160, 161, 191, 202, 211, 224–226, 233, 269, **397**, 398, 403
$\mathbb{R}$-simple set, **103**, 106, 133, 135, 136, 138, 201, 202
Refined Patching Lemma, **290**, 311, 314, 316–318
regular sequence
ψ_v-regular sequence, xxiii, **285**, 286–290, 292, 295–297, 299, 302–305, 308–310, 312–314, 316
repatch, 305–308
representation of U_v, **112**
Riemann surface, 6, 57, 76, 104, 133, 141, 351
Riemann, Bernhard, 11
Riemann-Roch theorem, 65, 66, 392, 399, 407
Riesz Decomposition theorem, 336, 377
rigid analytic function, 401
rigid analytic space, 5, 130, 398
Robin constant, xiii, **xvii**, 9, 193, 225, 336, 338–341, 346–348, 353, 386
$(\mathfrak{X}, \vec{s})$, xxiii, **332–336**, 338–341, 344–346, 348, 349, 370, 371, 373–377, 384–387
archimedean, 134–136, 145, 146
archimedean $(\mathfrak{X}, \vec{s})$, 137, 141, 146
bounds for, 147
properties of, 145, 150
nonarchimedean, 119, 159, 162, 165, 166, 172–174, 177, 178, 181, 182, 200–203
nonarchimedean $(\mathfrak{X}, \vec{s})$, 163–165, 167, 168, 172, 175–178, 182, 185
takes on rational values, **88**, **164**, 167, **345, 346**
computing nonarchimedean, 22, 23, 25–27, 45–49, 118, 164
Berkovich, **6**, **121**, 124, 129
compatible with classical, **125**, 126
monotonicity of, 127
properties, 130, 131
properties of, **120**, **121**, 122–125
classical, **xiii**
examples of Robin constants

archimedean, 9, 13, 14, 16, 17, 19, 20, 31, 35–38
nonarchimedean, 21, 22, 24, **25**, 26, 31, 32, 34, 35, 37, 117
on elliptic curves, 41, **42**, 43, 50–54
on Fermat curves, 55
on modular curves, 58
global, 6, 203, 206, 209, 225–227, 229
local, 208, 224, 226
of compact set, **78**, 125
properties of, 83, **88**, 104–108, 111, 113, 115, 118, 133, 135, 144 , 347
upper, xviii, 1, 2, **82**, **85**, **88**, 91, 133
upper global, 93
weighted $(\mathfrak{X}, \vec{s})$, **354**, 356, 366–370, 372–374
Robinson, Raphael, ix, xii, xv, xx, 31, 260
rock-paper-scissors argument, xxiii, 352
Rolle's theorem, 15
roots, 296, 313
in good position, 306
endangered roots, **306**, 311
safe roots, **306**, 311, 314, 318
logarithmically separated, 314, 316
long safe sequence of roots, 311, **312**
move roots, 297, 307–310, 316
natural one-to-one correspondence between roots in successive steps of patching, **295**
patched roots, **306**, 311, 314, 316, 318
unpatched roots, **306**, 310, 311, 314
roots of unity, 195
Roquette, Peter, xii
Rouché's theorem, 267
Rumely, Robert, xiii, 5, 120, 336

S-subunit, **196**
S-unit, **195**
S-unit Theorem, 195
Saff, Ed, 352
scaled isometry, **97**, 100, 162, 169–171, 176, 177, 188, 189, 201, 205, 224, 227, 228
power series map induces, **97**
Schmid, Joachim, xiii
Schwarz Reflection Principle, 12, 18
Schwarz-Christoffel map, 12
Sebbar, Ahmad, 12–14
Second Moving Lemma, **307**, **324**
see-saw argument, **218**, 221, 238, 240
semi-continuous
Green's function is upper semi-continuous, **82**, 85, **86**, 87, 124
potential function is lower semi-continuous, 355, 377
semi-local theory, 196–199
for number fields, 198
for function fields, 198
separate roots, 216, 233, 284, 305, 309, 311
Shifrin, Ted, xxiv, 154
Shimura, Goro, 11
'short' interval, xxiii, 136, 138, 144, **145**, 146, 147, 151, 351, 352, 378, **379–382**, 383, 384, 386, 387
simply connected, 76, 103, 105, 133, 202, 249, 257, 378, 383–387
size of an adele, **193**
skeleton of a Berkovich curve, 124
spherical metric, x, 45, 61, 62, **69, 70**, 71, 75, 78, 104, 105, 110, 117, 119, 144, 161, 171, 331, 355, 395, 400
continuity of, 114
from different embeddings comparable, **70**, 395
Galois equivariance of, 108
on curve, 407
on Jacobian, 179, 408
Stirling polynomial, xv, xxi–xxiii, 293
for $\mathcal{O}_v$, 170, 188, 211, 215, 231, 280, 284
high-order coefficients vanish, 293
when $\mathrm{char}(K_v) = p > 0$, 293
for $\mathcal{O}_w$, **98**, 99, 100
generalized, **171**, **172**, 177, 181, 182
Strong Approximation theorem , adelic, 191, **193**
Uniform Strong Approximation theorem, **194**, 212, 216, 220, 222, 236, 239, 241
subharmonic, 333
subunit, 192, **196**, 200, 205, 207, 209, 210
superharmonic, 333, 354, 355, 368, 377, 385
supersingular points, **57**, 58, 59
Szegö, Gábor, ix, xi, xii, 212, 260, 411
Szpiro, Lucien, xii

Tamagawa, Akio, xiii
tame curve, xx, **57**
Tate's algorithm, 50
Tate, John, 50
Teichmüller representatives, **98**, 294
terminal ray of a Newton polygon, 96, 97
theta-functions, xx
classical, 10, 11, 13, 35
of genus two, 14
thin set, **89**, 90
Third Moving Lemma, **309**, **327**
Thuillier, Amaury, 5, 120, 125, 126, 336
Totik, Vilmos, 352
transfinite diameter, xxiii, 352, 361
$(\mathfrak{X}, \vec{s})$, xxiii, 331, **332**
extended, xii
weighted $(\mathfrak{X}, \vec{s})$, 352, 362, 366, 372–374
triangulation, 104, 105

uniformizing parameter, **63**
Galois equivariant system, 63, 159
used to normalize L-rational basis, **65**, 66, 67, 224, 243, 244
used to normalize L^{sep}-rational basis, **66**

used to normalize canonical distance, **xvi**, 73, 74, **76**, 78, 81, 82, 125, 164, 345, 347
used to normalize capacity, 31, 41, 51–53
used to normalize Robin constant, **xviii**, 2, 9, 31, 33, 34, 42, 50, 54, 58, 60, **82**, 85, 88, 91, 122
units , totally real, ix, xii
universal function, xxiii, 319, 321, 324, **389**, 389–405
University of Georgia, xxiv
unpatched roots, 305

value of Γ as a matrix game, **xviii**, 31, 33, 104, 128, 130
van den Dries, Lou, xiii
Varley, Robert, xxiii, xxiv, 390

Weierstrass equation, xx, 50, 51
for specific elliptic curves, 40, 51–53
minimal, 42, 50–53
nonminimal, 52
Weierstrass Factorization theorem, 95
Weierstrass Preparation Theorem, 289, 291, 292, 411
weights
for nonarchimedean equilibrium distribution are rational, **164**
in the product formula, **61**
weights $\log(q_v)$ in $\Gamma(\mathbb{E},\mathfrak{X})$, **xviii**, **92**
Weil
distribution, 70
divisor, 390
height, 36
Weil, André, 65, 391, 407
well-adjusted model, 44
well-separated, 305
Widom, Harold, xx, 14, 16

$\mathfrak{X}$, viewed as embedded in $\mathcal{C}_v(\mathbb{C}_v)$, **62**
$\mathfrak{X}$-trivial, x, xi, xv, 2–4, 6, 7, 30, 31, 33, 35, 37–40, 54, 62, 63, 69, 91, 92, 103, 104, 110, 115–117, 119, 120, 127–130, 159, 160, 191, 192, 202, 213, 224, 225
$(\mathfrak{X},\vec{s})$-function, xxi, **77**–**78**, 136–139, 167, 168, 170, 175, 181, 188, 197–202, 204, 206, 213, 215, 235, 237, 239, 240, 242, 243, 246
K_v-rational, **133**, 134–136, 160–162, 166, 215, 221, 224, 225, 227, 228, 232, 233
$(\mathfrak{X},\vec{s})$-potential theory, **331**

Young, H. Peyton, 160