Index

abelian scheme, 14, 46, 47 dual, 36 isogeny, 37 polarization, 37 abelian variety absolutely simple, 15 and primary extension of fields, 15 isotypic, 22, 23, 31 lifting to characteristic 0, 86 Poincaré reducibility, 16 simple over a field, 15 abelian variety with sufficiently many complex multiplication, 24, 25, 77 Grothendieck's theorem in char. $p > 0$, 79 refinement, 86 admissible algebraic homomorphism, 301 and Weil numbers, 315 construction of, 316 field of moduli, 310 primitive, 299 weight of, 303 Albert algebra, 28 classification of, 29 algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127, 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 bad p -adic place, 208, 208, 209, 210, 246 pair, 209, 235, 236, 238	<i>a</i> -number, 143	Barsotti-Tate group, see also p-divisible
dual, 36 isogeny, 37 polarization, 37 abelian variety absolutely simple, 15 and primary extension of fields, 15 isotypic, 22, 23, 31 lifting to characteristic 0, 86 Poincaré reducibility, 16 simple over a field, 15 abelian variety with sufficiently many complex multiplication, 24, 25, 77 Grothendieck's theorem in char. $p > 0$, 79 refinement, 86 admissible algebraic homomorphism, 301 and Weil numbers, 315 construction of, 316 field of moduli, 310 primitive, 299 weight of, 303 Albert algebra, 28 classification of, 29 algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127, 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 bad p -adic place, 208, 208, 209, 210, 246 truncated, absorb truncated at level 1, 144 Brauer group, 18 of a global field, 20 order of an element of, 20 of a local field, 19 of a plobal field, 20 of a local field, 19 of a plobal field, 20 of a local field, 19 of a plobal field, 20 of a local field, 19 of a plobal field, 20 of a local field, 19 of a plobal field, 20 of a local field, 19 of a plobal field, 20 of a local field, 19 of a local field, 20 of a local field, 20 of a local field, 19 of a local field, 19 of a local field, 19 of a local field, 20 of a local field, 20 of a local field, 19 of a local field, 19 of a local field, 19 of a local field, 20 of a local field, 19 of a local field, 19 of a local field, 20 of a local field, 19 of a local field, 20 of a local field, 20 of a local field, 19 of a local field, 19 of a local field, 20 of a local field, 20 of a local field, 19 of a local field, 19 of a local field, 19 of a local field, 20 of	abelian scheme, 14, 46, 47	
polarization, 37 abelian variety absolutely simple, 15 and primary extension of fields, 15 isotypic, 22, 23, 31 lifting to characteristic 0, 86 Poincaré reducibility, 16 simple over a field, 15 abelian variety with sufficiently many complex multiplication, 24, 25, 77 Grothendieck's theorem in char. $p > 0$, 79 refinement, 86 admissible algebraic homomorphism, 301 and Weil numbers, 315 construction of, 316 field of moduli, 310 primitive, 299 weight of, 303 Albert algebraic Part of, 119 and complex multiplication, 127, 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 ap-adic place, 208, 208, 209, 210, 246 truncated at level 1, 144 Brauer group, 18 of a global field, 20 order of an element of, 20 of a local field, 19 order of an element of, 20 of a local field, 19 order of an element of, 20 of a local field, 19 order of an element of, 20 of a local field, 20 order of an element of, 20 of a local field, 19 order of an element of, 20 of a local field, 20 order of an element of, 20 of a local field, 20 order of an element of, 20 of a local field, 20 order of an element of, 20 ord	dual, 36	truncated, see also truncated
polarization, 37 abelian variety absolutely simple, 15 and primary extension of fields, 15 isotypic, 22, 23, 31 lifting to characteristic 0, 86 Poincaré reducibility, 16 simple over a field, 15 abelian variety with sufficiently many complex multiplication, 24, 25, 77 Grothendieck's theorem in char. $p > 0$, 79 refinement, 86 admissible algebraic homomorphism, 301 and Weil numbers, 315 construction of, 316 field of moduli, 310 primitive, 299 weight of, 303 Albert algebraic Part of, 119 and complex multiplication, 127, 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 ap-adic place, 208, 208, 209, 210, 246 truncated at level 1, 144 Brauer group, 18 of a global field, 20 order of an element of, 20 of a local field, 19 order of an element of, 20 of a local field, 19 order of an element of, 20 of a local field, 19 order of an element of, 20 of a local field, 20 order of an element of, 20 of a local field, 19 order of an element of, 20 of a local field, 20 order of an element of, 20 of a local field, 20 order of an element of, 20 of a local field, 20 order of an element of, 20 ord	isogeny, 37	Barsotti-Tate group
abelian variety absolutely simple, 15 and primary extension of fields, 15 isotypic, 22, 23, 31 lifting to characteristic 0, 86 Poincaré reducibility, 16 simple over a field, 15 abelian variety with sufficiently many complex multiplication, 24, 25, 77 Grothendick's theorem in char. $p > 0$, 79 refinement, 86 admissible algebraic homomorphism, 301 and Weil numbers, 315 construction of, 316 field of moduli, 310 primitive, 299 weight of, 303 Albert algebra, 28 classification of, 29 algebraic part of, 119 and complex multiplication, 127, 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α cgroup scheme, 142 Dieudonné module of, 358–359 Bad p -adic place, 208, 208, 209, 210, 246 Brauer group, 18 of a global field, 20 order of an element of, 20 order of an e		~ -
absolutely simple, ${\bf 15}$ and primary extension of fields, 15 isotypic, ${\bf 22}$, ${\bf 23}$, ${\bf 31}$ liffing to characteristic 0, ${\bf 86}$ Poincaré reducibility, ${\bf 16}$ simple over a field, ${\bf 15}$ abelian variety with sufficiently many complex multiplication, ${\bf 24}$, ${\bf 25}$, ${\bf 77}$ Grothendieck's theorem in char. $p>0$, $p=0$ refinement, $p=0$ refinement of, $p=0$ refinement, $p=0$ refineme	-	
and primary extension of fields, 15 isotypic, 22, 23, 31 lifting to characteristic 0, 86 Poincaré reducibility, 16 simple over a field, 15 abelian variety with sufficiently many complex multiplication, 24, 25, 77 Grothendieck's theorem in char. $p > 0$, 79 refinement, 86 admissible algebraic homomorphism, 301 and Weil numbers, 315 construction of, 316 field of moduli, 310 primitive, 299 weight of, 303 Albert algebra, 28 classification of, 29 algebraic part of, 119 and complex multiplication, 127, 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 and companding module of, 20 cartier theory, 351–353 Cartier module, 322, 352 V -reduced, 3	· ·	~
isotypic, 22 , 23 , 31 lifting to characteristic 0, 86 Poincaré reducibility, 16 simple over a field, 15 abelian variety with sufficiently many complex multiplication, 24 , 25 , 77 Grothendieck's theorem in char. $p > 0$, 79 refinement, 86 admissible algebraic homomorphism, 301 and Weil numbers, 315 construction of, 316 field of moduli, 310 primitive, 299 weight of, 303 Albert algebra, 28 classification of, 29 algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127 , 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, $358-359$ and the smooth formal group \widehat{W} , 351 central simple algebra, 16 Wedderburn's theorem, 17 CM abelian variety, 23 , 32 descent to a number field, 67 existence over field of moduli, 311 isotypic, 31 L-function of, 272 potential good reduction, 78 CM algebra, 26 CM field, 26 CM formal abelian scheme criterion for algebraicity, 106 CM lifting of a p -divisible group, 169 CM lifting questions, 86 after finite residue field extension (R), 88 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 strong CM lifting (sCML), 88 , 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100 ,		
lifting to characteristic 0, 86 Poincaré reducibility, 16 simple over a field, 15 abelian variety with sufficiently many complex multiplication, 24, 25, 77 Grothendieck's theorem in char. $p > 0$, 79 refinement, 86 admissible algebraic homomorphism, 301 and Weil numbers, 315 construction of, 316 field of moduli, 310 primitive, 299 weight of, 303 Albert algebra, 28 classification of, 29 algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127, 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 bad p -adic place, 208, 208, 209, 210, 246 Cartier dual, 34, 326 Cartier theory, 351–353 Cartier module, 322, 352 V -flat, 352 V -reduced, 352 Cartier ing, 351 and the smooth formal group \widehat{W} , 351 central simple algebra, 16 Wedderburn's theorem, 17 CM abelian variety, 23, 32 descent to a number field of moduli, 311 isotypic, 31 L-function of, 272 potential good reduction, 78 CM algebra, 26 CM field, 26 CM formal abelian scheme criterion for algebraicty, 106 CM lifting of a p -divisible group, 169 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,	isotypic, 22 , 23, 31	,
simple over a field, 15 abelian variety with sufficiently many complex multiplication, 24 , 25 , 77 Grothendieck's theorem in char. $p > 0$, 79 refinement, 86 admissible algebraic homomorphism, 301 and Weil numbers, 315 construction of, 316 field of moduli, 310 primitive, 299 weight of, 303 Albert algebra; 28 classification of, 29 algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127 , 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, $358-359$ bad p -adic place, 208 , 208 , 209 , 210 , 246	lifting to characteristic 0, 86	,
simple over a field, 15 abelian variety with sufficiently many complex multiplication, 24 , 25 , 77 Grothendieck's theorem in char. $p > 0$, 79 refinement, 86 admissible algebraic homomorphism, 301 and Weil numbers, 315 construction of, 316 field of moduli, 310 primitive, 299 weight of, 303 Albert algebra; 28 classification of, 29 algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127 , 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, $358-359$ bad p -adic place, 208 , 208 , 209 , 210 , 246	Poincaré reducibility, 16	Cartier dual 34 326
abelian variety with sufficiently many complex multiplication, 24 , 25 , 77 Grothendieck's theorem in char. $p > 0$, 79 refinement, 86 admissible algebraic homomorphism, 301 and Weil numbers, 315 construction of, 316 field of moduli, 310 primitive, 299 weight of, 303 Albert algebra, 28 classification of, 29 algebraic Hecke character, 28 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 24 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, 300 cogroup scheme,	simple over a field, 15	
complex multiplication, 24, 25, 77 Grothendieck's theorem in char. $p > 0$, 79 refinement, 86 admissible algebraic homomorphism, 301 and Weil numbers, 315 construction of, 316 field of moduli, 310 primitive, 299 weight of, 303 Albert algebra, 28 classification of, 29 algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127, 128 compatible system of ℓ-adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α-group scheme, 142 Dieudonné module of, 358–359 bad p -adic place, 208, 208, 209, 210, 246 V-flat, 352 V-reduced, 352 Cartier ring, 351 and the smooth formal group \widehat{W} , 351 central simple algebra, 16 Wedderburn's theorem, 17 CM abelian variety, 23, 32 descent to a number field, 67 existence over field of moduli, 311 isotypic, 31 L-function of, 272 potential good reduction, 78 CM algebra, 26 CM field, 26 CM fifting of a p -divisible group, 169 CM lifting questions, 86 after finite residue field extension (R), 88 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTT), 210 strong CML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,	abelian variety with sufficiently many	* '
Grothendieck's theorem in char. $p>0$, 79 refinement, 86 admissible algebraic homomorphism, 301 and Weil numbers, 315 construction of, 316 field of moduli, 310 primitive, 299 weight of, 303 Albert algebra, 28 classification of, 29 algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127, 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 bad p -adic place, 208, 208, 209, 210, 246 V-reduced, 352 Cartier ring, 351 and the smooth formal group \widehat{W} , 351 central simple algebra, 16 Wedderburn's theorem, 17 CM abelian variety, 23, 32 descent to a number field, 67 existence over field of moduli, 311 isotypic, 31 L-function of, 272 potential good reduction, 78 CM algebra, 26 CM field, 26 CM formal abelian scheme criterion for algebraicity, 106 CM lifting of a p -divisible group, 169 CM lifting questions, 86 after finite residue field extension (R), 88 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LITI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,	complex multiplication, 24, 25, 77	
refinement, 86 admissible algebraic homomorphism, 301 and Weil numbers, 315 construction of, 316 field of moduli, 310 primitive, 299 weight of, 303 Albert algebra, 28 classification of, 29 algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127, 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 to and the smooth formal group \widehat{W} , 351 central simple algebra, 16 Wedderburn's theorem, 17 CM abelian variety, 23, 32 descent to a number field, 67 existence over field of moduli, 311 isotypic, 31 L-function of, 272 potential good reduction, 78 CM algebra, 26 CM field, 26 CM formal abelian scheme criterion for algebraicity, 106 CM lifting of a p -divisible group, 169 CM lifting questions, 86 after finite residue field extension (R), 88 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,	Grothendieck's theorem in char. $p > 0$,	
refinement, 86 admissible algebraic homomorphism, 301 and Weil numbers, 315 construction of, 316 field of moduli, 310 primitive, 299 weight of, 303 Albert algebra, 28 classification of, 29 algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127, 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 and the smooth formal group \widehat{W} , 351 central simple algebra, 16 Wedderburn's theorem, 17 CM abelian variety, 23, 32 descent to a number field, 67 existence over field of moduli, 311 isotypic, 31 L-function of, 272 potential good reduction, 78 CM algebra, 26 CM field, 26 CM formal abelian scheme criterion for algebraicity, 106 CM lifting of a p -divisible group, 169 CM lifting questions, 86 after finite residue field extension (R), 88 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,	79	,
and Weil numbers, 315 construction of, 316 field of moduli, 310 primitive, 299 weight of, 303 Albert algebra, 28 classification of, 29 algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127, 128 compatible system of ℓ-adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula, for an algebraic Hecke character weight of, 300 α-group scheme, 142 Dieudonné module of, 358-359 and Weiderburn's theorem, 17 CM abelian variety, 23, 32 descent to a number field, 67 existence over field of moduli, 311 isotypic, 31 L-function of, 272 potential good reduction, 78 CM algebra, 26 CM field, 26 CM formal abelian scheme criterion for algebraicity, 106 CM lifting of a p-divisible group, 169 CM lifting questions, 86 after finite residue field extension (R), 88 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian variety, 23, 32 descent to a number field, 67 existence over field of moduli, 311 isotypic, 31 L-function of, 272 potential good reduction, 78 CM algebra, 26 CM fifting of a p-divisible group, 169 CM lifting (cML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,	refinement, 86	<u> </u>
and Weil numbers, 315 construction of, 316 field of moduli, 310 primitive, 299 weight of, 303 descent to a number field, 67 existence over field of moduli, 311 isotypic, 31 L-function of, 272 potential good reduction, 78 CM algebra; 28 classification of, 29 algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127, 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 cargoup scheme, 142 Dieudonné module of, 358–359	admissible algebraic homomorphism, 301	
construction of, 316 field of moduli, 310 primitive, 299 weight of, 303 Albert algebra, 28 classification of, 29 algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127, 128 compatible system of ℓ-adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α-group scheme, 142 Dieudonné module of, 358–359 bad p-adic place, 208, 208, 209, 210, 246 CM abelian variety, 23, 32 descent to a number field, 67 existence over field of moduli, 311 isotypic, 31 L-function of, 272 potential good reduction, 78 CM algebra, 26 CM formal abelian scheme criterion for algebraicity, 106 CM lifting of a p-divisible group, 169 CM lifting questions, 86 after finite residue field extension (R), 88 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTT), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,	and Weil numbers, 315	
primitive, 299 weight of, 303 Albert algebra, 28 classification of, 29 algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127, 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 descent to a number field, 67 existence over field of moduli, 311 isotypic, 31 L-function of, 272 potential good reduction, 78 CM algebra, 26 CM formal abelian scheme criterion for algebraicity, 106 CM lifting of a p -divisible group, 169 CM lifting questions, 86 after finite residue field extension (R), 88 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,	construction of, 316	•
primitive, 299 weight of, 303 Albert algebra, 28 classification of, 29 algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127, 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 existence over field of moduli, 311 isotypic, 31 L-function of, 272 potential good reduction, 78 CM algebra, 26 CM field, 26 CM formal abelian scheme criterion for algebraicity, 106 CM lifting of a p -divisible group, 169 CM lifting questions, 86 after finite residue field extension (R), 88 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,	field of moduli, 310	***
weight of, 303 Albert algebra, 28 classification of, 29 algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127, 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 isotypic, 31 L-function of, 272 potential good reduction, 78 CM algebra, 26 CM field, 26 CM formal abelian scheme criterion for algebraicity, 106 CM lifting of a p-divisible group, 169 CM lifting questions, 86 after finite residue field extension (R), 88 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,		,
classification of, 29 algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127, 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 bad p -adic place, 208, 208, 209, 210, 246 L-function of, 272 potential good reduction, 78 CM algebra, 26 CM field, 26 CM formal abelian scheme criterion for algebraicity, 106 CM lifting of a p -divisible group, 169 CM lifting questions, 86 after finite residue field extension (R), 88 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,	9 ,	•
classification of, 29 algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127, 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 bad p -adic place, 208, 208, 209, 210, 246 potential good reduction, 78 CM algebra, 26 CM field, 26 CM formal abelian scheme criterion for algebraicity, 106 CM lifting of a p -divisible group, 169 CM lifting questions, 86 after finite residue field extension (R), 88 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,	-	
algebraic Hecke character, 118 algebraic part of, 119 and complex multiplication, 127, 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 bad p -adic place, 208, 208, 209, 210, 246 CM algebra, 26 CM field, 26 CM formal abelian scheme criterion for algebraicity, 106 CM lifting of a p -divisible group, 169 CM lifting questions, 86 after finite residue field extension (R), 88 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,	·	
and complex multiplication, 127, 128 compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 bad p -adic place, 208, 208, 209, 210, 246 CM field, 26 CM formal abelian scheme criterion for algebraicity, 106 CM lifting of a p -divisible group, 169 CM lifting questions, 86 after finite residue field extension (R), 88 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,		
compatible system of ℓ -adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 bad p -adic place, 208, 208, 209, 210, 246 CM formal abelian scheme criterion for algebraicity, 106 CM lifting of a p -divisible group, 169 CM lifting questions, 86 after finite residue field extension (R), 88 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,		
compatible system of t-adic characters attached to, 121 construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 to module of, 358–359 existence theorem, 142 condition, 102 condition, 1		•
construction by surgery procedure, 124 equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 Ead p -adic place, 208, 208, 209, 210, 246 CM lifting of a p -divisible group, 169 CM lifting questions, 86 after finite residue field extension (R), 88 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,	- v	
equivalent definition, 120 existence over field of moduli, 311 existence theorem, 305 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 bad p -adic place, 208, 208, 209, 210, 246 of a p -divisible group, 169 CM lifting questions, 86 after finite residue field extension (R), 88 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,		~
existence over field of moduli, 311 existence theorem, 305 after finite residue field extension (R), 88 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 abelian schemes isogenous to a CM lift of (LTI), 210 to normal domains up to isogeny (IN), 88 bad p-adic place, 208, 209, 210, 246 CM lifting questions, 86 after finite residue field extension (R), 88 after finite residue field extension (R), 88 after finite residue field extension (R), 88 cM Lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,		9
existence over lead of moduli, of 1 after finite residue field extension (R), 88 formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 abelian schemes isogenous to a CM lift (LTI), 210 to normal domains up to isogeny (IN), 88 bad p-adic place, 208, 209, 210, 246 after finite residue field extension (R), 88 after finite residue field extension (R), 88 cM (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,	,	
formula for, see Shimura Taniyama formula, for an algebraic Hecke character weight of, 300 α -group scheme, 142 Dieudonné module of, 358–359 bad p -adic place, 208, 208, 209, 210, 246 CM lifting (CML), 87 sufficient condition, 102 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,	•	
formula, for an algebraic Hecke character Lie type of the closed fibers of CM weight of, 300 abelian schemes isogenous to a CM lift α -group scheme, 142 (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 bad local method, 178 p-adic place, 208, 209, 210, 246 necessary and sufficient condition, 100,		` '
character weight of, 300 abelian schemes isogenous to a CM lift α -group scheme, 142 (LTI), 210 to normal domains up to isogeny (IN), 88 bad p-adic place, 208, 209, 210, 246 Lie type of the closed fibers of CM abelian schemes isogenous to a CM lift (LTI), 210 to normal domains up to isogeny (IN), 88 local method, 178 necessary and sufficient condition, 100,		- , , , , , , , , , , , , , , , , , , ,
weight of, 300 abelian schemes isogenous to a CM lift α -group scheme, 142 (LTI), 210 strong CM lifting (sCML), 88, 210 to normal domains up to isogeny (IN), 88 bad local method, 178 p-adic place, 208, 209, 210, 246 necessary and sufficient condition, 100,		Lie type of the closed fibers of CM
$\begin{array}{c} \alpha\text{-group scheme, } 142 & \text{(LTI), } 210 \\ \text{Dieudonn\'e module of, } 358\text{-}359 & \text{strong CM lifting (sCML), } 88, 210 \\ \text{to normal domains up to isogeny (IN), } 88 \\ \text{bad} & \text{local method, } 178 \\ p\text{-adic place, } 208, 208, 209, 210, 246 & \text{necessary and sufficient condition, } 100, \end{array}$		* -
Dieudonné module of, 358–359 strong CM lifting (sCML), 88 , 210 to normal domains up to isogeny (IN), 88 local method, 178 p-adic place, 208, 208 , 209, 210, 246 necessary and sufficient condition, 100,	9 ,	(LTI), 210
bad to normal domains up to isogeny (IN), 88 local method, 178 p-adic place, 208, 209, 210, 246 necessary and sufficient condition, 100,		strong CM lifting (sCML), 88, 210
p-adic place, 208, 208, 209, 210, 246 necessary and sufficient condition, 100,		to normal domains up to isogeny (IN), 88
· · · · · · · · · · · · · · · · · ·	bad	local method, 178
pair, 209, 235 , 236, 238 128–136	p-adic place, 208, 208, 209, 210, 246	necessary and sufficient condition, 100,
	pair, 209, 235 , 236, 238	128–136

386 INDEX

to normal domains up to isogeny after	of a cocharacter, 315
finite residual field extension (RIN)	field of definition as obstruction to CML,
Honda-Tate theorem on (RIN), 75	181–185
to normal domains up to isogeny after	field of moduli
finite residue field extension (RIN), 88	of an admissible algebraic
up to isogeny (I), 88	homomorphism, 310
existence, 195	r)
CM order, 65, 68, 80, 89	good
CM p-divisible group, 167	<i>p</i> -adic place, 208, 208 , 209, 247
existence, 172, 178	pair, 209, 235 , 238
Galois representation of, 176	Grothendieck group, 207, 208, 211, 212,
-	227, 241, 324
uniqueness up to isogeny, 173 CM structure	,
	Hodge-Tate decomposition, 168, 185, 189
dual, 67, see also CMtype of the dual of	Honda-Tate theorem, 71
a CM abelian variety	and CM lifting, 3
for abelian varieties, 2, 32	G/
CM type	Kisin modules, 360–362
for a CM algebra, 66	
for a CM field, 66	level structure
of a CM abelian variety, 66	finite étale, 13, 57
determines the isogeny class, 67	Lie type, 212 , 229
of the dual of a CM abelian variety, 67	and Galois descent, 207, 211, 217
p-adic, see p-adic CM type	rational over a field, 215 , 236, 237, 242
valued in a field, 66	self-dual, 230 , 235, 236, 239, 242, 242
co-Lie complex, 356	striped, 208 , 234, 234 , 236, 238, 239
counterexample	local deformation space
to (CML) and (R), 183–184	for a CM structure, see deformation ring,
to (IN), 101, 110	for a CM structure
with two slopes, 111–114	of a p -divisible group, 169
to CM lifting with action by the full ring	
of integers in the CM field, 198	main theorem of complex multiplication,
crystalline Dieudonné theory, 354–359	127, 257-292
	algebraic form, 266
deformation ring	analytic form, 288
for a CM structure, 202	converse to, 128, 292–296
of a CM structure, 92	multiplicaties of a Lie type, 233
of a p -divisible groups, 169	
deformation ring argument, 91, 106, 169,	Newton polygon, 114
233	and (IN), 114–116
deformation theory	ander lettice 286
for abelian schemes, 196	order lattice, 286
for p -divisible groups, 196	p-adic abelian crystalline representation,
Dieudonné theory, 38, 137–138, 347–351	174, 189
basic differential invariants, 356–359	algebraic on the inertia subgroup, 190
comparison of Dieudonné theories, 356	p-adic CM type, 168
Dieudonné ring, 38	compatible with a given CM structure,
Dieudonné-Manin classification, 139	170
dimension of a Lie type, 212	of a CM p -divisible group, 168, 206
display, 354, 362–368	self-dual, 206 , 208, 209, 230 , 232, 235,
duality theorem	238, 239, 242
for abelian varieties, 37	<i>p</i> -adic Hodge theory, 333, 359
for p -divisible groups, 152	
	p-divisible group, 39
effective elements	a-number of, 143
in $R_k(\mathcal{O}_F)$ and $R_{\kappa}(\mathcal{O}_F)$, 211 , 212, 229	and deformation of abelian varieties, 59
extended Lubin-Tate type, 156	connected, 40, 41 Dioudonné Manin classification up to
field of definition	Dieudonné-Manin classification up to
of a p-adic cocharacter, 187	isogeny, 139
or a p-aute contraracter, 101	étale, 40 , 140

INDEX 387

height of, 39 isoclinic, 139 isogeny of, 148–152 local-local, 140 local-local part, 140 of multiplicative type, 140 ordinary, 140 quasi-polarization of, 60 Serre dual of, 39 slopes of, see slopes, of a p-divisible group with sufficiently many complex multiplication, 169	and residual reflex condition, 100 for a CM abelian variety, 98 for an algebraic Hecke character, 313 short exact sequence, 34 singular <i>j</i> -invariant, 1 Skolem–Noether theorem, 18 slopes of a Lie type, 214 , 217, 233, 233 of a <i>p</i> -divisible group, 139, 139 , 207, 215, 237, 322 of an abelian variety, 98 striped Lie type, see Lie type, striped supersingular <i>j</i> -values, 1
polarization, 37	
reciprocity laws sign conventions of, 10 reduction map from p-adic CM types to Lie types, 212, 230 reflex field of a CM type, 93 of a p-adic cocharacter, 187 of a p-adic CM type, 170, 172, 182, 183 reflex norm of a CM type, 94, 95 of a p-adic cocharacter, 187, 189 of a p-adic CM type, 171 of a p-adic CM type, 171 of a p-adic cocharacter, 188 replete divisor, 308, 309 degree of, 308 principle, 308 residual reflex condition, 100 necessary and sufficient condition for (IN), 100, 128–136	Tate's theorem and (RIN), 75 CM structure for abelian varieties with sufficiently many complex multiplication, 26 extending homomorphisms between p-divisible groups, 58 Hodge-Tate decomposition for p-divisible groups, 58 homomorphisms between abelian varieties over a finite field, 70 toy model, 110 CM abelian surface, 196, 203 classification of, 220 CM p-divisible group, 204, 209, 236–239 higher dimensional, 321, 333 truncated Barsotti-Tate group BT1 group scheme, 144 BTn group, 143
self-dual Lie type, 230 , 232, 235, 236, 239,	uniform Lie type, 233 , 235, 242, 242 , 244
242, 242	Weil q-integer, 70
self-dual p -adic CM type, 206 , 208, 209, 230 , 232, 235, 238, 239, 242 self-dual up to isogeny, 243 , 244 self-duality and algebraization, 243 and reduction modulo p , 231 condition, 199, 206, 208, 209, 228, 230, 243, 244, 248	Weil q -number of weight w , 70 , 304, 313, 315 existence, 318 slopes of, 315 Weil restriction of scalars, 22 windows, 362 Witt covectors, 345 , 345–347
Serre group attached to a number field, 120 neutral component, 119 , 299 character group of, 299 weight cocharacter of, 303	
weight cocharacter of, 303 Serre tensor product construction and Lie types, 227 for abelian varieties, 80, 83, 224 for p-divisible groups, 238 for p-divisible groups, 224, 236 Serre-Tate canonical lifting, 60 Serre-Tate theorem on deformation of	

abelian varieties, 59, 206 Shimura–Taniyama formula, 98

and (IN), 100