

Preface

During the *Workshop on Abelian Varieties* in Amsterdam in May 2006, the three authors of this book formulated two refined versions of a problem concerning lifting into characteristic 0 for abelian varieties over a finite field. These problems address the phenomenon of *CM lifting*: the lift into characteristic 0 is required to be a CM abelian variety (in the sense defined in 1.3.8.1). The precise formulations appear at the end of Chapter 1 (see 1.8.5), as problems (I) and (IN).

Abelian surface counterexamples to (IN) were found at that time; see 2.3.1–2.3.3, and see 4.1.2 for a more thorough analysis. To our surprise, the same counterexamples (typical among *toy models* as defined in 4.1.3) play a crucial role in the general solution to problems (I) and (IN). This book is the story of our adventure guided by CM lifting problems.

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