

# Contents

Preface	vii
Chapter 1. Basic concepts	1
1.1. General notation	1
1.2. Words over an alphabet	2
1.3. Combinatorial spaces	4
1.4. Reduced and extracted words	6
1.5. Carlson–Simpson spaces	7
1.6. Trees	11
1.7. Notes and remarks	14
<b>Part 1. Coloring theory</b>	<b>15</b>
Chapter 2. Combinatorial spaces	17
2.1. The Hales–Jewett theorem	17
2.2. The multidimensional Hales–Jewett theorem	22
2.3. Colorings of combinatorial spaces	25
2.4. Notes and remarks	34
Chapter 3. Strong subtrees	37
3.1. The Halpern–Läuchli theorem	37
3.2. Milliken’s tree theorem	42
3.3. Homogeneous trees	51
3.4. Notes and remarks	55
Chapter 4. Variable words	57
4.1. Carlson’s theorem	57
4.2. Applications	60
4.3. Finite versions	64
4.4. Carlson–Simpson spaces	72
4.5. Notes and remarks	73
Chapter 5. Finite sets of words	75
5.1. Subsets of combinatorial spaces	75
5.2. Subsets of Carlson–Simpson spaces	80
5.3. Notes and remarks	86
<b>Part 2. Density theory</b>	<b>87</b>
Chapter 6. Szemerédi’s regularity method	89
6.1. Decompositions of random variables	89

6.2. Szemerédi's regularity lemma	99
6.3. A concentration inequality for product spaces	102
6.4. Notes and remarks	107
Chapter 7. The removal lemma	109
7.1. A multidimensional version of Theorem 6.5	110
7.2. A regularity lemma for hypergraph systems	112
7.3. A counting lemma for hypergraph systems	114
7.4. Proof of Theorem 7.2	124
7.5. Applications	127
7.6. Notes and remarks	132
Chapter 8. The density Hales–Jewett theorem	133
8.1. Sperner's theorem	133
8.2. Preliminary tools	134
8.3. Proof of Theorem 8.1	137
8.4. Applications	144
8.5. Notes and remarks	153
Chapter 9. The density Carlson–Simpson theorem	155
9.1. The convolution operation	155
9.2. Iterated convolutions	161
9.3. Some basic estimates	164
9.4. A probabilistic version of Theorem 9.2	172
9.5. An exhaustion procedure: achieving the density increment	180
9.6. Proof of Theorem 9.2	198
9.7. Proof of Theorem 9.1	199
9.8. Applications	201
9.9. Notes and remarks	208
<b>Part 3. Appendices</b>	211
Appendix A. Primitive recursive functions	213
Appendix B. Ramsey's theorem	215
Appendix C. The Baire property	217
Appendix D. Ultrafilters	219
Appendix E. Probabilistic background	227
Appendix F. Open problems	233
Bibliography	237
Index	243