

# Contents

Preface	vii
Chapter 1. Introduction	1
Chapter 2. Topology and Analysis	5
2.1. Euclidean $n$ -space	5
2.2. Möbius $n$ -space	6
2.3. Recollections from linear algebra	7
2.4. Dilatation and distortion of linear maps	11
2.5. Partial derivatives	11
2.6. Differentiability	12
2.7. Maximal and minimal stretchings	14
2.8. Diffeomorphisms	14
Chapter 3. Conformal Mappings in Euclidean Space	17
3.1. Linear conformal transformations	17
3.2. Reflections	20
3.3. The Möbius group	23
3.4. Hyperbolic geometry	37
3.5. Classification of hyperbolic isometries	47
3.6. The distortion, compactness and convergence properties of Möbius transformations	49
3.7. The Möbius group as a matrix group	56
3.8. Liouville's theorem	64
Chapter 4. The Moduli of Curve Families	77
4.1. Path integrals	87
4.2. Moduli of curve families	99
4.3. Technical properties of moduli	125
4.4. Extremal metrics	140
4.5. ACL-functions and Fuglede's theorem	143
Chapter 5. Rings and Condensers	151
5.1. Rings	151
5.2. Condensers	160
5.3. Spherical symmetrization of condensers	167
5.4. Estimating the moduli of rings	180
5.5. Sets of capacity zero	182
5.6. Extremal functions for condensers	184

Chapter 6. Quasiconformal Mappings	205
6.1. The definition of quasiconformality via conformal moduli	205
6.2. Examples and the computation of dilatation	210
6.3. Some measure theory	222
6.4. The analytic characterisation of quasiconformality	229
6.5. The boundary behavior of quasiconformal mappings	251
6.6. The distortion, compactness and convergence properties of quasiconformal families	271
6.7. Quasiconformal mappings of $\mathbb{H}^n$ with the same boundary values	298
6.8. The 1-quasiconformal mappings	300
Chapter 7. Mapping Problems	307
7.1. Existence of extremal mappings	309
7.2. Topological obstructions: Wild bilipschitz spheres	310
7.3. Geometric obstructions to existence	314
7.4. Existence: The Schoenflies theorem	323
7.5. Väisälä's theorem on cylindrical domains	334
7.6. Quasiconformal homogeneity	351
Chapter 8. The Tukia-Väisälä Extension Theorem	355
8.1. Lipschitz embeddings	356
8.2. Preliminaries	362
8.3. The Tukia-Väisälä extension theorem	371
Chapter 9. The Mostow Rigidity Theorem and Discrete Möbius Groups	381
9.1. Introduction and statement of the theorem	381
9.2. Hyperbolic manifolds, covering spaces and Möbius groups	384
9.3. Quasiconformal manifolds and quasiconformal mappings	388
9.4. Quasi-isometries	390
9.5. Groups as geometric objects	393
9.6. The boundary values are quasiconformal	398
9.7. The limit set of a Möbius group	402
9.8. Mappings compatible with a Möbius group	409
9.9. The proof of Mostow's theorem	412
Basic Notation	417
Bibliography	419
Index	427