

## Preface

This book is the result of an intended research paper that grew out of control. A preprint containing a substantial part of our investigations was already published on arXiv in 2010. To make its content more accessible, we decided to include some additional material. These additions more than doubled the size of this work as compared with the 2010 version and caused a long delay in its completion.

More than fifteen years ago we became both interested in some basic problems on quasisymmetric parametrization of 2-spheres. This is related to the dynamics of rational maps—an observation we believe was first made by Rick Kenyon. During our time at the University of Michigan we decided to join forces and to investigate this connection systematically.

We realized that for the relevant rational maps an explicit analytic expression is not so important, but rather a geometric-combinatorial description. As this became our preferred way of looking at these objects, it was a natural step to consider a more general class of maps that are not necessarily holomorphic. The relevant properties can be condensed into the notion of an *expanding Thurston map*, which is the topic of this book. We will discuss the underlying ideas more thoroughly in the introduction (Chapter 1).

Part of this work overlaps with studies by other researchers, notably Haïssinsky-Pilgrim [HP09], and Cannon-Floyd-Parry [CFP07]. We would like to clarify some of the interrelations of our investigations with these works. Theorem 15.1 (in the body of the text) was announced by the first author during an Invited Address at the AMS Meeting at Athens, Ohio, in March 2004, where he gave a short outline of the proof. After the talk he was informed by Bill Floyd and Walter Parry that related results had been independently obtained by Cannon-Floyd-Parry (which later appeared as [CFP07]).

Theorem 18.1 (ii) was previously published by Haïssinsky-Pilgrim as part of a more general statement [HP09, Theorem 4.2.11]. Special cases go back to work by the second author [Me02] and unpublished joint work by Bruce Kleiner and the first author. The current, more general version emerged after a visit of the first author at the University of Indiana at Bloomington in February 2003.

During this visit the first author explained to Kevin Pilgrim concepts of quasi-conformal geometry and his joint work with Bruce Kleiner on Cannon's conjecture in geometric group theory. Kevin Pilgrim in turn pointed out Theorem 11.1 and the ideas for its proof to the first author. After this visit versions of Theorem 18.1 (ii) with an outline for the proof were found independently by Kevin Pilgrim and the first author. A proof of Theorem 18.1 (ii) was discovered soon afterwards by the authors using ideas from [Me02] (see [Me10] for an argument along similar lines) in combination with Theorem 15.1.

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