Introduction

The aim of this book is to give a self-contained presentation of recent mathematical and computational advances in photonics and phononics. The fields of photonics and phononics encompass the fundamental science of light and sound propagation and interactions in complex structures, and its technological applications.

The recent advances in nanoscience present great challenges for the applied and computational mathematics community. In nanophotonics, the aim is to control, manipulate, reshape, guide, and focus electromagnetic waves at nanometer length scales, beyond the resolution limit. In particular, one wants to break the resolution limit by reducing the focal spot and confine light to length scales that are significantly smaller than half the wavelength. Nanostructures also present exciting opportunities for designing acoustic material properties for specific applications.

Interactions between the fields of photonics and phononics and mathematics have led to the emergence of a multitude of new and unique solutions in which today's conventional technologies are approaching their limits in terms of speed, capacity and accuracy. Light and sound can be used for detection and measurement in a fast, sensitive and accurate manner, and thus photonics and phononics are uniquely positioned to revolutionize healthcare. Light-and sound-based technologies can be used effectively for the very early detection of diseases, with non-invasive imaging techniques or point-of-care applications. They are also instrumental in the analysis of processes at the molecular level, giving a greater understanding of the origin of diseases, and hence allowing prevention along with new treatments.

The main objective in this book is to report on the use of sophisticated mathematics in diffractive optics, plasmonics, super-resolution, photonic and phononic crystals, Helmholtz resonators and Minnaert bubbles, and metamaterials for electromagnetic and elastic invisibility and cloaking. The book merges highly nontrivial multi-mathematics in order to make a breakthrough in the field of mathematical modelling, imaging, and design of optical and acoustic nanodevices and nanostructures capable of light or sound enhancement, and of the focusing and guiding of light or sound at a subwavelength scale. We demonstrate the power of layer potential techniques in solving challenging problems in photonics and phononics, when they are combined with asymptotic analysis and the elegant theory of Gohberg and Sigal on meromorphic operator-valued functions.

In this book we shall consider both analytical and computational matters in photonics and phononics. The issues we consider lead to the investigation of fundamental problems in various branches of mathematics. These include asymptotic analysis, spectral analysis, mathematical imaging, optimal design, stochastic modelling, and analysis of wave propagation phenomena. Deriving mathematical foundations, and new and efficient computational frameworks and tools in photonics and phononics, requires a deep understanding of the different scales in the wave propagation problem, an accurate mathematical modelling of the nanodevices, and fine analysis of complex wave propagation phenomena.

The book is divided into five parts. In Part 1, we introduce our main mathematical and computational tools for analyzing photonic and phononic problems. Chapter 1 is devoted to the theory of characteristic values of operator-valued functions developed by Gohberg and Sigal. Chapter 2 offers a comprehensive treatment of layer potential techniques. In Chapter 3, we provide a general and unified integral equation approach based on the Gohberg-Sigal theory for deriving asymptotic expansions of eigenvalue perturbations. The main idea is to reduce the eigenvalue problems to the study of characteristic values of systems of certain integral operators.

In Part 2, we consider scattering problems by periodic structures. Chapter 4 is devoted to the analysis of scattering from diffractive gratings. In Chapters 5 and 6, we investigate the bandgap structure of the spectrum for waves in a high contrast, two-component periodic medium. We provide tools for studying photonic and phononic crystals based on layer potentials.

In Part 3, we consider direct and inverse scattering from sub-wavelength resonators. We show that plasmon resonant nanoparticles provide a possible means of achieving super-resolved imaging. We study the resonant property of high-contrast particles for different particle geometries and environments, and use them to achieve super-resolution. In Chapter 7, we introduce a layer potential formulation for plasmonic resonances and derive asymptotic formulas for the plasmonic resonances and the scattered fields in terms of the particle size. We derive an effective medium theory for suspensions of plasmonic nanoparticles. We prove that at or near plasmonic resonances, the effective medium possesses a high contrast. We also consider field enhancement for a system of plasmonic spheres and investigate plasmonic resonances for domains with corners. In Chapter 8, we discuss direct imaging of small particles. We make use of small-volume expansions for the scattered fields to design efficient non-iterative imaging algorithms. We show that the resolution limit, defined as the minimum distance required between two small particles to distinguish between them, is of the order of half the operating wavelength. In Chapter 9, we introduce super-resolution techniques in order to differentiate point sources or small particles which are located less than half the wavelength apart. We present the mathematical foundation for realizing super-resolution by using high-contrast or resonant media. The key idea is to design the surrounding medium so that the imaginary part of the corresponding Green's function is tailored in such a way as to allow us to resolve sub-wavelength details.

In Part 4, we consider the cloaking of waves and metasurfaces. In Chapter 10, we focus on interior cloaking and describe effective near-cloaking structures. In Chapter 11, the focus is placed on exterior cloaking. We link the exterior cloaking issue to the existence of anomalous localized resonance, which is associated to an infinite dimensional kernel of a non-elliptic operator. In Chapter 12, we describe the anomalous reflection properties of a thin layer of periodic plasmonic nanoparticles.

In Part 5, we consider sub-wavelength acoustic resonators. We show that Helmholtz resonators and bubbles have the potential to be basic building blocks not only for sub-wavelength acoustic imaging but also for acoustic metamaterials. In Chapter 13, we prove that the spectrum of the Helmholtz resonator essentially coincides with the spectrum of the Laplacian with Neumann boundary condition in the closed cavity, but there is an additional resonant frequency which is a subwavelength resonance, which is the key to super-resolution for acoustic waves in systems of Helmholtz resonators. In Chapter 14, we consider acoustic wave propagation in bubbly media. A distinctive feature of bubbles in fluid is the high contrast between the air density inside and outside of the bubble. This results in a quasi-static acoustic resonance, called the Minnaert resonance. Using Minnaert resonances, we prove that super-focusing properties, bandgap openings at the subwavelength scale, and double-negative refractive index phenomena can be obtained for systems of bubbles.

The bibliography provides a list of relevant references. It is by no means comprehensive. However, it should provide the reader with some useful guidance in searching for further details on the main ideas and approaches discussed in this book.

The material in this book is taught as a graduate course in applied mathematics at ETH Zürich. The Matlab computer codes used in both the book and the course can be downloaded at the official supplementary materials website for the book which is located at `www.ams.org/bookpages/surv-235`.

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