

# Contents

Introduction	1
<b>Part 1. Mathematical and Computational Tools</b>	<b>5</b>
Chapter 1. Generalized Argument Principle and Rouché's Theorem	7
1.1. Introduction	7
1.2. Argument Principle and Rouché's Theorem	7
1.3. Definitions and Preliminaries	8
1.4. Factorization of Operators	11
1.5. Main Results of the Gohberg and Sigal Theory	13
1.6. Muller's Method	16
1.7. Concluding Remarks	17
Chapter 2. Layer Potentials	19
2.1. Introduction	19
2.2. Sobolev Spaces	19
2.3. Layer Potentials for the Laplace Equation	21
2.4. Neumann–Poincaré Operator	23
2.5. Conductivity Problem in the Free Space	38
2.6. Periodic and Quasi-Periodic Green's Functions	54
2.7. Shape Derivatives of Layer Potentials	64
2.8. Layer Potentials for the Helmholtz Equation	68
2.9. Laplace Eigenvalues	76
2.10. Helmholtz-Kirchhoff Identity, Scattering Amplitude and Optical Theorem	86
2.11. Scalar Wave Scattering by Small Particles	98
2.12. Quasi-Periodic Layer Potentials for the Helmholtz Equation	105
2.13. Computations of Periodic Green's Functions	108
2.14. Integral Representation of Solutions to the Full Maxwell Equations	120
2.15. Integral Representation of Solutions to the Lamé System	141
2.16. Quasi-Periodic Layer Potentials for the Lamé System	166
2.17. Concluding Remarks	168
Chapter 3. Perturbations of Cavities and Resonators	169
3.1. Introduction	169
3.2. Optical Cavities	169
3.3. Optical Resonators	182
3.4. Elastic Cavities	183

3.5. Eigenvalue Perturbations Due to Shape Deformations	190
3.6. Concluding Remarks	191
<b>Part 2. Diffraction Gratings and Band-Gap Materials</b>	<b>193</b>
Chapter 4. Diffraction Gratings	195
4.1. Introduction	195
4.2. Electromagnetic Theory of Gratings	195
4.3. Variational Formulations	203
4.4. Boundary Integral Formulations	221
4.5. Optimal Design of Grating Profiles	223
4.6. Numerical Implementation	224
4.7. Concluding Remarks	226
Chapter 5. Photonic Band Gaps	227
5.1. Introduction	227
5.2. Floquet Transform	228
5.3. Structure of Spectra of Periodic Elliptic Operators	228
5.4. Boundary Integral Formulation	229
5.5. Sensitivity Analysis with Respect to the Index Ratio	239
5.6. Photonic Band Gap Opening	248
5.7. Sensitivity Analysis with Respect to Small Perturbations in the Geometry of the Holes	249
5.8. Proof of the Representation Formula	250
5.9. Characterization of the Eigenvalues of $\tilde{\Delta}$	252
5.10. Maximizing Band Gaps in Photonic Crystals	252
5.11. Photonic Cavities	254
5.12. Concluding Remarks	255
Chapter 6. Phononic Band Gaps	257
6.1. Introduction	257
6.2. Asymptotic Behavior of Phononic Band Gaps	258
6.3. Criterion for Gap Opening	275
6.4. Gap Opening Criterion When Densities Are Different	278
6.5. Concluding Remarks	280
<b>Part 3. Subwavelength Resonant Structures and Super-resolution</b>	<b>281</b>
Chapter 7. Plasmonic Resonances for Nanoparticles	283
7.1. Introduction	283
7.2. Quasi-Static Plasmonic Resonances	284
7.3. Effective Medium Theory for Suspensions of Plasmonic Nanoparticles	286
7.4. Shift in Plasmonic Resonances Due to the Particle Size	290
7.5. Plasmonic Resonance for a System of Spheres	299
7.6. Quasi-Static Plasmonic Resonances for Domains with Corners	306
7.7. Concluding Remarks	313
Chapter 8. Imaging of Small Particles	315
8.1. Introduction	315

8.2. Scalar Wave Imaging of Small Particles	315
8.3. Electromagnetic Imaging	319
8.4. Elasticity Imaging	319
8.5. Numerical Illustrations	322
8.6. Concluding Remarks	322
Chapter 9. Super-Resolution Imaging	325
9.1. Introduction	325
9.2. Super-Resolution Imaging in High-Contrast Media	325
9.3. Super-Resolution in Resonant Structures	336
9.4. Super-Resolution Based on Scattering Tensors	350
9.5. Concluding Remarks	351
<b>Part 4. Metamaterials</b>	<b>353</b>
Chapter 10. Near-Cloaking	355
10.1. Introduction	355
10.2. Near-Cloaking in the Quasi-Static Limit	356
10.3. Near-Cloaking for the Helmholtz Equation	359
10.4. Near-Cloaking for the Full Maxwell Equations	367
10.5. Near-Cloaking for the Elasticity System	374
10.6. Concluding Remarks	381
Chapter 11. Anomalous Resonance Cloaking and Shielding	383
11.1. Introduction	383
11.2. Layer Potential Formulation	385
11.3. Anomalous Resonance in an Annulus	387
11.4. Shielding at a Distance	393
11.5. Concluding Remarks	400
Chapter 12. Plasmonic Metasurfaces	403
12.1. Introduction	403
12.2. Setting of the Problem	404
12.3. Boundary-Layer Corrector and Effective Impedance	405
12.4. Numerical Illustrations	409
12.5. Concluding Remarks	409
<b>Part 5. Sub-wavelength Phononics</b>	<b>413</b>
Chapter 13. Helmholtz Resonator	415
13.1. Introduction	415
13.2. Hilbert Transform	415
13.3. Perturbations of Scattering Frequencies of a Helmholtz Resonator	420
13.4. Resonances of a System of Helmholtz Resonators and Super-Resolution	424
13.5. Concluding Remarks	427
Chapter 14. Minnaert Resonances for Bubbles	429
14.1. Introduction	429
14.2. Derivation of Minnaert Resonance Formula	430

14.3.	Effective Medium Theory for a System of Bubbles and Super-Resolution	438
14.4.	Sub-wavelength Phononic Bandgap Opening	453
14.5.	Double-Negative Refractive Index Phenomenon	461
14.6.	Numerical Illustrations	464
14.7.	Concluding Remarks	471
Appendix A.	Spectrum of Self-Adjoint Operators	473
Appendix B.	Optimal Control and Level Set Representation	477
B.1.	Optimal Control Scheme	477
B.2.	Level Set Method	477
B.3.	Shape Derivatives	479
Bibliography		481
Index		507