

Preface

This is neither a book on Lie algebras nor on Jordan systems. This book is just intended to show how Jordan theory can be applied to Lie algebras. In one sense it is introductory, since very few prerequisites on Lie algebras are presupposed. In fact, Lie readers, especially those accustomed to working with root systems or with the fruitful relationship between Lie groups and Lie algebras, might be disappointed with the fact that most of the Lie algebras considered here are over a ring of scalars where only the invertibility of 2, 3, and sometimes 5, is assumed, and even in the case of Lie algebras over a field, these are not necessarily finite-dimensional. Yet in this poor soil a germinal form of root system can still grow: the grading induced by a compatible family of idempotents (a notion which is defined in Jordan terms and which connects with the Jacobson–Morozov lemma) resembles in a broad sense the root decomposition associated to a root system. On the other hand, the book is far from being elementary. We allow ourselves to use any Jordan result under the sun if this were required for solving a single Lie question.

Honestly I must say that in spite of the abundance of results that the reader can find on applications of Jordan theory to Lie algebras in the text, the book has some shortcomings, as the lack of a proof, or at least an outline of it, of the structure theorem of simple Lie algebras with a finite grading. Nevertheless, making a virtue of necessity, this important gap might encourage him to study the proof directly from Zelmanov’s paper itself.

This book is the fruit of a long collaboration of the author with Esther García and Miguel Gómez Lozano; a great deal of the results exposed in it are due to them. I am also grateful to Alexander Baranov, Jose Brox, Miguel Cabrera, Esperanza Sánchez Campos, and Guillermo Vera De Salas for their valuable suggestions and again to Miguel Gómez Lozano for his help and technical assistance in \TeX questions.

I acknowledge the support of the Spanish Ministerio de Economía y Competitividad ¹

I hope that the reader can enjoy reading the book as much as I did writing it.

Antonio Fernández López
Málaga, Spain
October, 2018

¹MTM2013-47154-P, MTM2014-52470-P, and MTM2017-84194-P (AEI/FEDER, UE.)