

## Preface

In this book, the reader can find the theory of autonomous and non-autonomous perturbations for attractors of dynamical systems, from the coarsest notion of proximity - upper semicontinuity - to the finest notion of proximity - geometric structural stability. We assume that the reader only has some basic knowledge of metric and Banach spaces in what concerns the theoretical part of the book. For the applications the reader will be required to master the theory of semilinear evolution equations in general spaces and some partial differential equations.

The book is divided into two parts: Part I - Autonomous Theory and Part II - Non-autonomous Theory. Each part has five chapters and they are correlated between Part I and Part II, that is, Chapter 1 has its analogue for the non-autonomous case in Chapter 6, and so on. For the autonomous theory of Part I, Chapter 1 contains the basic definitions and results regarding semigroups and their global attractors. Chapter 2 deals with upper and lower semicontinuity. Chapter 3 deals with dynamically gradient semigroups and the topological structural stability of their attractors. Chapter 4 studies neighborhoods of critical elements, that is, equilibrium points and periodic orbits, and presents the proof of the  $\lambda$ -lemma of J. Palis but in the infinite-dimensional case. This proof was found in D. Henry's handwritten notes "Invariant Manifolds near a Fixed Point", and many other results of this chapter were taken from it as well. Chapter 5 presents the concept of Morse-Smale semigroups and the geometrical structural stability of their attractors, that is, an isomorphism between their phase diagrams. In Part II, as we said, the chapters are the analogues of their counterparts in the autonomous theory. Several results from Part II are new developments of the theory that, we hope, will contribute to further understanding the asymptotics of non-autonomous problems.

We want to express our gratitude to Eriko Hironaka at AMS who, throughout the writing of this book, has been very supportive and patient. Also, we thank the anonymous reviewers for the support and suggestions that led to many improvements. We also wish to thank Yanan Li, Estefani Moraes Moreira and Alexandre N. Oliveira-Sousa who read the entire manuscript and made several suggestions during the preparation of it.

In May 2019 we lost our collaborator and warm-hearted friend Geneviève Raugel. With Geneviève we developed the results on stability of the phase diagram isomorphisms for Morse-Smale semigroups under non-autonomous perturbations, presented in the second part of this book. We wish to honour her at this opportunity.

Matheus C. Bortolan  
Alexandre N. Carvalho  
José A. Langa