Preface

Inspired by earlier work exhibiting v_1-periodicity in the topological cyclic homology of the integers [**30**], [**31**], [**148**], [**149**], and subsequent work exhibiting v_2-periodicity in the topological cyclic homology of the connective complex K-theory ring spectrum and its Adams summand [**19**], [**18**], the authors started an investigation into the topological Hochschild homology and topological cyclic homology of the topological modular forms ring spectrum, aiming to study the v_3-action on $F_*TC(tmf)$ for suitable finite type 3 spectra F. In particular, at the prime $p = 2$ we can take F to be the homotopy cofiber of a map $v_2^{32} \colon \Sigma^{192} M(1,4) \to M(1,4)$ as in [**26**], and then $F \wedge tmf \simeq tmf/(2, B, M)$ for certain Bott and Mahowald elements $B \in \pi_8(tmf)$ and $M \in \pi_{192}(tmf)$.

The Adams spectral sequence, in conjunction with the computer software package `ext` described in [**41**], provides a flexible and powerful tool for making calculations with *tmf*, $THH(tmf)$ and approximations to $TC(tmf)$. The additive structure of the Adams spectral sequence for *tmf*, and parts of its multiplicative structure, have been known to Mahowald and some other experts for many years [**76**], [**54**, Ch. 13], but for our project we expect to need full information about the multiplicative structure. Since we believe that this detailed information will be of use and interest also to other researchers in algebraic topology, we have composed the following account of the Adams spectral sequence for *tmf*, and related spectra such as $tmf/(2, B, M)$, aiming to give complete information and proofs of results that have otherwise mostly been available as folklore.

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