

Preface

This book is a largely expanded version of a course that the second author gave at ICTP in Trieste in the summer 2012, preceding a conference on “hypergeometric motives”, in Rennes in April 2014 at the Journées Louis Antoine, and at the NTCS workshop in Taiwan in August 2014.

The goal of this book is to present a number of analytic and arithmetic numerical methods used in number theory, with a particular emphasis on the ones which are less known than they should be, although very classical tools are also mentioned. Note that, as is very often the case in number theory, we want numerical methods giving sometimes *hundreds* if not thousands of decimal places of accuracy. The typical timing tables that we will give are in fact for 500 decimal digits.

The style of presentation is the following: we first give proofs of some of the tools, the prerequisites being classical undergraduate analysis. Note that since the emphasis is on *practicality*, the proofs are sometimes only heuristic, but valid in actual practice. We then give the corresponding `Pari/GP` programs, usually followed by a number of examples. These programs are also available as a unique separate archive on the authors’ website at

http://www.math.u-bordeaux.fr/~kbelabas/Numerical_Algorithms/

Feel free to experiment and modify them to your heart’s content. They can also serve as an introduction to the syntax and semantics of `GP`, since in general they are easy to understand and do not use much sophistication. Note we use rather recent features of the `GP` language, so we strongly advise to download the latest release (version 2.13 or more recent) from the `Pari/GP` website:

<http://pari.math.u-bordeaux.fr/>

The reader is advised to refer to the numerous books dealing with parts of the subject, such as (but of course not limited to!) the second author’s four books [Coh93], [Coh00], [Coh07a], and [Coh07b].

Caveat: Neither the authors nor the AMS are liable for any damage caused by the use of the programs given in this book. Apart from this legalese, we would be happy to hear of any corrections and/or improvements. Even though we have tried to give simple and efficient programs, we do not claim that we give “the best” methods, and we would also be very glad to hear of new methods for solving the problems considered in this book.

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