

# Index

- Abhyankar, 1, 38, 109, 113, 163, 166, 231, 386, 391
- Affine algebraic variety, 104
- Affine ring, 104, 109
- Akizuki, 1–3, 17, 49, 141
- Algebra
  - étale, 111, 142
  - essentially finite, 11
  - essentially of finite type, 11
  - finite, 11
  - finite type, 11
  - quasi-normal, 141
  - Rees, 140
- Algebraic independence, 12, 87
- Algebraic retract, 43
- Almost Cohen-Macaulay module, 256
- Alonzo-Tarrio, 375, 376
- Alonzo-Tarrio, Jeremias-Lopez and Lipman
  - Question: formal schemes, 375, 376
- Analytically independent, 386
- Analytically irreducible Noetherian local ring, 33, 49
- Analytically normal Noetherian local ring, 33, 49, 113, 154
- Analytically ramified Noetherian local ring, 34
- Analytically reducible Noetherian local ring, 34, 48, 50
- Analytically unramified Noetherian local ring, 33, 34, 109, 141
- Anderson, Dan, 180
- Annihilator ideal, of an element, 11
- Approximation domain, 3–6, 59, 60, 62, 63, 70, 71, 73, 81, 83, 84, 118, 127, 131, 136, 137, 143, 150, 152, 154, 174, 183, 186, 189, 190, 198, 200, 201, 206, 209, 211, 213, 215, 237, 239, 307, 402
  - Homomorphic Image Construction*, 213
  - Inclusion Construction*, 61
- Approximation methods
  - Homomorphic Image Construction*, 211
- Approximation theorem for Krull domains, 15
- Associated prime ideal, 11, 206, 244
  - embedded, 11
- Auslander, 14
- Avramov, 244, 246
- Ax, 106, 154, 158, 162
- Azumaya, 36
  
- Benson, 381
- Berger, 113, 374
- Birational
  - domination, 10, 46, 150, 180, 181, 210, 239, 242
  - extension, 1, 7, 10, 209, 211, 218, 229, 235
- Bourbaki, 15, 117
- Brewer, 125, 204, 206
- Brodmann, 2, 4, 210, 231, 258, 261, 262
- Buchsbaum, 14
- Burch, 283, 330
  
- Catenary ring, 5, 7, 25, 34, 41, 69, 70, 85, 173, 175, 176, 186, 224, 231, 232, 234–236, 240, 242, 244
- Cauchy sequence, 28
- Chain condition for prime ideals, 232
- Chain Conjecture, 232, 246, 252
- Change of coefficient ring, 20
- Change of variables, 345, 346, 359, 365
- Charters, 341
- Chase, 206
- Chevalley, 18, 32, 109, 397
  - Theorem: Every Noetherian local domain is birationally dominated by a DVR, 18
- Christel's example, 51, 73, 137
- Classical examples, 2, 5
- Closed ideal
  - in the  $I$ -adic topology, 25, 28, 32, 42
- Closed singular locus, 109
- Coefficient field for a local ring, 33
- Cohen, 17, 23, 32, 33, 43, 109, 152, 177, 179, 180, 231, 342, 399
  - Structure theorems for complete local rings, 33, 342

- Theorem: A ring is Noetherian if each prime ideal is finitely generated, 17, 152
- Theorem: Every complete Noetherian local domain is a finite integral extension of a complete regular local domain, 33
- Theorem: Every complete Noetherian local ring is a homomorphic image of a complete regular local ring, 33
- Theorem: Every equicharacteristic complete Noetherian local ring has a coefficient field  $k$  and is a homomorphic image of a formal power series ring over  $k$ , 33
- Cohen's Theorem 8, 32
- Cohen-Macaulay  
fibers, 6, 88, 89  
formal fibers, 245, 249, 250  
ring, 35, 87, 87, 88, 101, 110, 140, 144
- Coherence, 204
- Coherent regular ring, 5, 206
- Coherent ring, 206
- Coherent sequences, 27, 30, 257–261
- Commutative diagram, 259, 260, 263
- Complete intersection, 10, 144, 246
- Completely integrally closed, 15
- Completion, 2, 5–7  
 $I$ -adic, 25  
associated to a filtration, 25  
coherent sequences, 259  
ideal-adic, 2, 3, 25, 26, 150, 257  
multi-adic, 257, 257, 258, 262, 265, 269, 271  
multi-ideal-adic, 6, 7, 257  
uncountable, 30
- Compositum of two power series rings, 154
- Conrad, 375, 377  
Question: formal schemes, 375, 376
- Constructed ring, 1, 2
- Construction  
*Homomorphic Image*, 209, 211, 213, 216, 237, 239, 240, 245  
*limit-intersecting*, 213  
*Inclusion*, 59, 60, 65, 73, 78, 81, 127, 209, 210, 220, 229, 245, 401  
*limit-intersecting*, 63  
*Insider*, 5, 81, 127, 130, 131, 401, 402  
*Intersection*, 2–4, 59, 401  
*limit-intersecting*, 115, 309  
*primarily limit-intersecting*, 310  
*residually limit-intersecting*, 309  
Remark: Every Inclusion Construction is up to isomorphism a Homomorphic Image Construction, 220
- Construction Properties Theorem, 3, 59, 64, 66, 68, 69, 128, 131, 137, 138, 145, 153, 175, 189, 191, 198, 213, 216, 217, 221, 237, 241, 272, 309, 311
- Contraction of an ideal, 35
- Cosupport, 381  
of a ring, 381  
of a finitely generated module, 381, 381
- Cutkosky, 166
- D+M construction, 184, 205, 207
- David, 173
- Delta ideal, 92
- Depth  
of a module, 38, 244  
of a ring, 38, 232, 244, 256
- Derivation, 92, 106
- Derived categories of chain complexes, 381
- Derived normal ring, 12, 232, 242, 244
- Differential morphism, 92
- Dilatation of  $R$  by  $I$  along  $V$ , 165
- Dimension  
of a prime ideal, 236  
of a ring, 10
- Dimension formula, 167, 225
- Direct limit of subrings, 3, 4, 47, 47
- Direct limits, 25
- Directed union of subrings, 3, 47
- Discrete rank-1 valuation domain, 7, 11, 165  
as a directed union of RLRs, 167
- Distinguished monic polynomial, 343, 344, 356
- Divisor Class Group, 16
- Divisorial ideal, 15
- Dominates, for local rings, 10, 42
- Dumitrescu, 41, 376, 386, 389
- DVR, 11, 13–15, 17, 21, 41, 47, 49, 55, 127–129, 156, 165, 167, 168, 170, 172, 175, 177, 180, 181, 184, 218, 223, 240, 348, 349, 358, 360–362, 364, 367, 370
- Eakin, 18, 163
- Eakin-Nagata, 18  
Theorem: If  $B$  is a Noetherian ring and  $A$  is a subring such that  $B$  is a finitely generated  $A$ -module, then  $A$  is a Noetherian ring, 18
- Eisenbud, 105
- Element of a ring  
prime, 10  
regular element, 9  
zerodivisor, 9
- Element of extension ring  
*ideally independent*, 275, 276, 277, 280, 285, 293, 294  
*limit-intersecting*, 3, 309  
*primarily independent*, 277, 277, 280, 283, 285, 287, 294, 294  
existence of, 283, 293

- residually algebraically independent*, 154, 277, 277, 280, 285–287, 289, 293, 294
- Elkik, 92
- Elkik ideal, 92, 93, 94, 105, 253
- Embedded associated prime ideal, 11
- Embedded local uniformization of  $R$  along  $V$ , 166, 167
- Endpiece notation, 60, 67, 128, 150, 174, 188, 190, 200, 212, 220, 221, 223
- Endpiece Recursion Relation, 61
- Equicharacteristic local ring, 33, 342
- Equidimensional ring, 6, 34, 85, 228, 234, 235, 240
  - formally, 233
- Essential valuation rings for a Krull domain, 15, 122, 125
- Essentially finite extension of commutative rings, 11
- Essentially finitely generated over a field, 355, 371, 372
- Essentially finitely generated over a commutative ring, 11, 370
- Essentially finitely presented over a commutative ring, 11, 91
- Essentially of finite type extension, 11
- Essentially smooth morphism, 91, 93, 95, 96
- Etale, 111
  - algebra, 111, 142
  - extension, 111, 169, 170, 302
  - homomorphism, 111
  - neighborhood, 111, 112
- Evans, 182
- Example
  - a non-catenary four-dimensional local UFD with maximal ideal generated by three elements that has precisely one prime ideal of height three and this prime ideal is not finitely generated, 200
  - an injective local map of normal Noetherian local domains that satisfies  $LF_{d-1}$  but not  $LF_d$ , 295
  - a 2-dimensional Noetherian local domain whose generic formal fiber is not Cohen-Macaulay, 250
  - a 3-dimensional RLR  $(A, \mathfrak{n})$  having a height-two prime ideal  $I = (f, g)A$  for which the extension  $I\hat{A}$  to the  $\mathfrak{n}$ -adic completion is not integrally closed, 5, 142
  - a non-universally catenary 2-dimensional Noetherian local domain with geometrically regular formal fibers that birationally dominates a 3-dimensional regular local domain, 240
  - a non-universally catenary 2-dimensional Noetherian local domain with geometrically regular formal fibers that birationally dominates a 3-dimensional regular local domain., 404
  - a regular local ring over a non-perfect field  $k$  that is not smooth over  $k$ , 106
  - a residually algebraically independent element need not be primarily independent, 287
  - an excellent ring that fails to satisfy Jacobian criteria, 106
  - an idealwise independent element may fail to be residually algebraically independent, 290
  - where the constructed domains  $A$  and  $B$  are equal and are not Noetherian, 5, 135
  - where the constructed domains  $A$  and  $B$  are not equal but  $A$  is Noetherian, 3, 5
  - where the constructed domains  $A$  and  $B$  are not Noetherian and are not equal, 248
- Examples
  - classical, 2, 5
  - Discrete valuation rings that are not Nagata rings, 129
  - iterative, 149
  - Non-catenary 3-dimensional local UFDs with maximal ideal generated by two elements that have  $m$  prime ideals of height two and each prime ideal of height two not finitely generated, 5, 173
  - Non-universally catenary  $n$ -dimensional Noetherian local domains with geometrically regular formal fibers, 5, 231
    - related to integral closure, 2
- Excellence, 5–7, 40, 41, 104, 106, 107, 109, 110, 127, 130, 136, 263
  - preservation of under completion, 265
- Excellent ring, 2, 3, 5–7, 25, 40, 50, 51, 84, 103, 104, 106, 109, 110, 113, 127, 129, 130, 136, 137, 145, 153, 156, 222, 224, 229, 240, 245, 252, 255, 257, 265, 293, 302, 327, 330–332, 340, 341
- Exponential functions, 154
- Extension
  - trivial generic fiber/TGF, 369, 373
- Extension of commutative rings
  - height-one preserving*, 115
  - weakly flat*, 115
  - essentially finite, 11
  - essentially finitely generated, 11
  - essentially finitely presented, 11
  - essentially of finite type, 11
  - finite, 11
  - finite type, 11
  - height-one preserving, 123

- integral closure in, 12
- integral extension, 12
- $LF_d$ , 115, 294
- $LF_{d-1}$ , 123, 134, 135
- Extension of integral domains
  - birational, 1, 10, 60, 209–211
  - trivial generic fiber/TGF, 6, 342, 377–379, 385, 386, 389
- Extension of Krull domains
  - height-one preserving, 276, 308
  - locally flat in height  $r$ , or  $LF_r$ , 308
  - weakly flat, 276, 308, 309
  - PDE, 15, 276, 286, 307
- Extension of local rings
  - étale, 111, 169, 170, 302
  - birationally dominates, 10
  - domination, 10
- Extensions of Krull domains
  - a weakly flat extension is height-one preserving, 119
  - height-one preserving and PDE imply weakly flat, 121
  - height-one preserving does not imply weakly flat, 120
  - PDE does not imply height-one preserving, 121
  - PDE does not imply weak flatness, 121
  - PDE is equivalent to  $LF_1$ , 121
  - weakly flat does not imply PDE, 121, 122
- Factorial ring, 10
- Faithful flatness, 18, 19, 37, 132, 137, 262, 264, 265, 350, 399
- Faithfully flat module, 18, 26
- Ferrand, 2
- Fiber ring, 36
- Fibers, 1, 88, 89, 234
  - Cohen-Macaulay, 6, 88, 245, 328
  - formal, 339, 340, 341, 371
  - generic, 339–341, 355, 371
  - geometrically normal, 5, 7, 39, 231, 244
  - geometrically reduced, 39
  - geometrically regular, 5, 7, 39, 42, 231, 232, 239, 240, 243, 340
  - normal, 39
  - of a ring homomorphism, 36
  - reduced, 39
  - regular, 39, 88, 89, 340
  - trivial generic fiber, 6
- Filtration of ideals, 257
  - $I$ -adic, 25
  - ideal-adic, 25
  - multi-adic, 257
  - multiplicative, 25, 257
  - not multiplicative, 257
- Finite
  - algebra extension, 11
  - module, 11, 370, 374
  - ring extension, 11
- Finite conductor domain, 206
- Finite presentation, for an algebra
  - extension, 11, 91
- Finite type, for an algebra extension, 11
- Flat extension, 4–6, 18, 375, 379
  - conditions for, 83, 87
- Flat homomorphism, 19, 20, 105, 189, 375, 379
- Flat locus
  - of a polynomial ring extension, 87, 97
- Flat module, 18, 37
- Flatness, 4–6, 18, 19, 22, 30, 40, 42, 73, 74, 81–83, 87, 89, 100, 101, 129, 130, 154, 211, 215, 217, 233, 237, 238, 293, 350, 351
  - elementwise criterion, 19
  - localization, 19
  - relations to the Jacobian ideal, 95
- Formal fibers, 5, 339, 340, 341, 371
  - Cohen-Macaulay, 246, 249, 250
  - for a Noetherian local ring, 40
  - geometrically normal, 5, 112, 113, 231, 233, 234, 244
  - geometrically reduced, 109, 112
  - geometrically regular, 110, 112, 113, 231, 232, 239, 240, 243, 340, 404
- Formal power series ring, 13, 22
  - leading form, 13
- Formal scheme, 375
- Formal spectrum, 376
- Formally equidimensional Noetherian local ring, 233
- Fossum, 15
- Fractional ideal, 15
- Frontpiece notation, 211, 212, 213, 220, 222, 223, 229
- Fuchs, 262
  
- G-ring, 40, 40, 136, 265
- Gabelli, 184, 205
- Gamma function, 157
- $\gcd(a, b)$ , 21
- GCD-domain, 21
- Generalization, stable under, 232, 235
- Generalized local ring, 180, 180
- Generalized power series, 170, 171
- Generic fiber, 6, 398
  - trivial generic fiber, 6, 342, 378
- Generic fiber ring, 340
  - mixed polynomial-power series rings, 339, 355
- Generic formal fiber, 6, 322, 355, 371
  - Theorem: height of maximal ideals, 341
- Generic formal fiber ring, 339, 340, 340, 361, 373
- Geometrically normal fibers, 39, 40

- Geometrically normal formal fibers, 5, 7, 233, 234
- Geometrically reduced fibers, 39, 40, 109
- Geometrically regular fibers, 5, 7, 39, 39, 41, 42, 105, 108, 109
- Geometrically regular formal fibers, 41, 232, 239, 240, 265, 340, 404
- Gff, 355, 370, 371, 373
- Gff (R), 339, 340, 340, 373
- Gilmer, 113, 204, 375
- Glaz, 5, 206
- Going-down property, 19, 95, 120, 232, 236, 249, 331, 337, 352
- Going-up Theorem, 183, 235, 358, 360
- Goto, 144
- Grade
  - of a module, 38
  - of a ring, 38, 256
- Graded ring, with respect to an ideal, 217
- Granja, 166
- Greatest common divisor, 21
- Greco, 240
- Griffith, 391
- Grothendieck, 103, 106, 109, 110, 234, 240
  
- Hartshorne, 104
- Hausdorff, 150, 151
- Height of a prime ideal, 10
- Height-one preserving, 115, 119–121, 123, 124, 276
- Height-one prime ideals are radicals of
  - principal ideals, 117, 122–124
- Heinzer, 46, 113, 163, 181, 340, 376, 380, 389, 399
- Heitmann, 2, 4, 38, 77, 210, 232, 246, 258, 261, 262, 340
  - How to adjoin a transcendental element preserving an ideal-adic completion, 4
  - Theorem: Every complete Noetherian local ring  $(T, \mathfrak{n})$  of depth at least two such that no nonzero element in the prime subring of  $T$  is a zerodivisor on  $T$  is the completion of a Noetherian local UFD, 232
- Hensel's Lemma, 36, 111
- Henselian affine partially ordered set, 380
- Henselian ring, 6, 25, 36, 37, 103, 111, 111, 169, 233, 257, 265, 267, 278, 302, 374
- Henselization, 5, 37, 103, 111, 111–113, 142, 169, 170, 231, 233–235, 242, 243, 267, 291, 293, 302
- Hilbert Nullstellensatz, 97
- Hironaka, 109
- Hochster, 6, 36, 257, 342, 377, 391
- Hochster and Yao
  - Question: trivial generic fibers, 342, 377
- Homomorphic image
  - of a regular local ring, 35, 240
- Homomorphic Image Construction, 209, 211, 213, 216, 220, 237, 240, 402
  - Approximation domain, 213
  - approximation methods, 211
- Homomorphism
  - étale, 111
  - essentially smooth, 91
  - flat, 19, 19, 20
  - local, 10
  - normal, 40
  - regular, 40, 51, 91, 101, 137, 234, 240
  - smooth, 91
- Houston, 184, 204, 205
- Huckaba, 140
- Huneke, 46, 139, 140
- Hypertranscendental element, 157
  
- Ideal
  - annihilator of an element, 11
  - closed in the  $I$ -adic topology, 25, 28, 32, 42
  - completion, 262
  - contraction of, 35
  - element integral over, 139, 140
  - extension in a ring homomorphism, 36
  - filtration, 25, 26, 257–259, 261, 263
  - integral closure, 139, 140
  - integrally closed, 139, 140, 140–143, 145–147
  - Jacobian, 146
  - normal, 139, 140, 140
  - order function, 13
  - radical, 146, 147
  - reduction, 139, 140, 140, 142
  - topology, 262, 268
- Ideal transforms, 125
- Ideal-adic
  - completion, 2, 3, 25, 26, 30, 37, 45, 48, 72, 150, 209, 220, 257, 265–267, 375, 376
  - filtration, 25
  - ideal separated for, 87
  - topology, 1, 25, 32, 37, 87, 150, 371, 376, 378
- Idealwise independent, 6, 275, 276–280, 285, 287, 293, 295, 298, 300, 303, 315
- Igusa, 163
- Immersion of schemes, 375, 376
  - closed, 375, 376
  - open, 375, 376
- Inclusion Construction, 59, 60, 60, 65, 70, 73, 78, 81, 127, 153, 155, 186, 209, 209, 210, 211, 220, 229, 307, 401
  - Approximation Domain, 61
- Independence
  - idealwise independence, 6, 275, 276, 278, 280, 285, 287, 293, 295, 298, 300, 303

- primary independence*, 6, 277, 280, 283, 285, 287, 293, 302, 305
- residual algebraic independence*, 154, 277, 278, 280, 285–287, 289, 293
- algebraic, 12, 87
- analytic, 386, 387
- Insider Construction, 401
- Insider Construction, 5, 73, 82, 87, 127, 130, 131, 133, 136, 139, 173, 186, 197, 402, 404
- Insider Inclusion Construction, 73
- Integral closure, 2, 12, 233
  - examples related to, 2
  - of an ideal, 139, 140
- Integral domain, 10
  - integral closure, 12
  - integrally closed, 5, 12
  - Krull domain, 14
  - regular local ring, 2, 3, 5, 14
  - unique factorization domain, 10
  - valuation domain, 11
- Integral element, 12
- Integral extension of commutative rings, 12
- Integral over an ideal, 139, 140
- Integrally closed domain, 5, 12, 140
- Integrally closed ideal, 139, 140, 141–143, 145–147
- Intersection Construction, 2–4, 6, 59, 209, 401
  - approximation domain, 3
  - equation, 4
  - intersection domain, 3
  - universality, 2, 46
- Intersection domain, 2, 3, 6, 46, 59, 60, 61, 62, 63, 81, 84, 118, 130, 131, 136, 154, 174, 189, 198, 200, 209, 210, 219, 238, 240, 307, 402
  - Noetherian limit, 76, 127, 225
- Inverse limits, 25
- Ionescu, 256
- Iterative examples, 149, 153, 154
- Iyengar, 381
- J-2 property of a Noetherian ring, 266
- Jacobian criterion, 106
  - for smoothness, 103, 105, 106
- Jacobian ideal
  - of a polynomial map, 87, 94, 95, 98, 136–138, 145, 146
- Jacobian matrix, 92, 106, 253
- Jacobson radical of a ring, 11, 16, 18, 26, 29, 61, 257–261, 264
- Jacobson ring, 340
- Jeremias-Lopez, 375, 376
- Kang, 256
- Katz, 34, 49
- Kearnes, 380
- Kiehl, 103, 109, 113, 374
- Kim, Youngsu, 371
- Krause, 381
- Kravitz, 38
- Krull, 16, 17, 162, 204, 231, 392
- Krull Altitude Theorem, 17, 22, 35, 392
- Krull dimension, 10
- Krull domain, 14, 14, 15, 17, 21, 23, 50, 117, 118, 149–151, 154, 199, 293, 307, 308, 310, 312, 325, 382
  - essential valuation rings, 15, 122
- Krull Intersection Theorem, 16, 162
- Krull-Akizuki theorem, 17, 46
- Kunz, 113, 154, 163, 374
  - Theorem: The ring compositum of  $k[[x]]$  and  $k[[y]]$  in  $k[[x, y]]$  is not Noetherian, 154, 163
- $\text{lcm}(a, b)$ , 21
- Least common multiple, 21
- Lequain, 2
- Leuschke, 184, 205, 283, 330
- $\text{LF}_d$ , 115, 123, 134, 135, 294, 308, 310–315, 324
- Limit-intersecting, 3, 63, 115, 309, 309, 310, 315
  - primarily, 310
  - residually, 309
  - for *Homomorphic Image Construction*, 213
  - for *Inclusion Construction*, 63
- Limits
  - Direct, 25
  - inverse, 25
- Linear topology, 25
- Linearly disjoint field extension, 149
- Lipman, 30, 141, 142, 375–377
- Local Flatness Theorem, 73, 192
- Local Homomorphic Image Prototype, 240
- Local homomorphism, 10
- Local Prototype, 54, 55, 73, 77, 82, 83, 137, 138, 153, 174, 239, 316
- Local Prototype Theorem, 130, 138, 142, 145, 153, 174, 175, 190, 200, 223, 240, 316
- Local quadratic transform, 49, 165, 166–169, 171, 184, 203, 399
  - iterated, 165
- Local ring, 10
  - étale neighborhood, 111
  - coefficient field, 33
  - coherent regular, 5, 206
  - equicharacteristic, 33
  - Henselian, 6, 36, 37, 233, 267
  - Henselization, 5, 37
  - regular local ring, 14
- Local uniformization of  $R$  along  $V$ , 166
- Localization, 9
- Localized polynomial ring, 10, 275

- Loepp, 147, 340, 341  
 Lost prime ideal, 178, 179–181, 183, 184  
 Luroth, 163  
 Luroth theorem, 163
- MacLane, 171  
 Marley, 114  
 Marot, 373  
 Matijevic, 180  
 Matlis, 262  
 Matsumura, 9, 20, 22, 91, 162, 266, 340, 341, 351, 356, 371, 391  
   Question: dimension of generic formal fiber in excellent local rings, 340  
   Theorem: dimension of generic formal fiber rings, 340, 355, 356
- McAdam, 2, 206, 373  
 Minimal prime divisor, 17  
 Mixed polynomial-power series rings, 1, 6, 339, 342, 355, 371, 373, 375, 385, 389  
   relations among spectra, 379  
   spectra, 375, 379
- Module  
    $I$ -adically ideal-separated, 87  
   elementwise criterion for flatness, 19  
   faithfully flat, 18  
   finite, 11  
   flat, 18, 19, 20  
   regular sequence on, 38  
   separated for the  $I$ -adic topology, 87, 87  
   torsionfree, 20
- Moh, 386, 391  
 Monoidal transform, 166  
 Multi-adic completion, 257, 257, 258, 262, 265, 269, 271  
   Noetherian flatness, 269  
 Multi-adic Inclusion Construction, 272  
 Multi-ideal-adic completion, 6, 7, 257, 262  
 Mumford, 18, 104, 111
- Nagata, 1, 2, 4, 16–18, 50, 73, 81–83, 103, 106, 108–111, 113, 137, 141, 151, 210, 231, 233  
   Example: a 2-dimensional analytically reducible normal Noetherian local domain, 50, 82  
   Example: a 2-dimensional regular local ring that is not a Nagata ring, 50  
   Example: a 2-dimensional regular local ring that is not a Nagata ring, 2, 4, 82, 137  
   Example: a 2-dimensional regular local ring with a prime element that factors as a square in the completion, 51, 82
- Nagata domain, 2, 51, 108, 129, 262  
 Nagata Polynomial Theorem, 16, 109  
 Nagata ring, 16, 51, 84, 103, 109, 110, 113  
 Nakayama's lemma, 32  
 Nastold, 103, 113, 374
- Neeman, 381  
 Nested union of subrings, 3, 47  
 Nichols, 180  
 Nilradical of a ring, 10  
 Nishimura, 2, 18, 23, 110, 258, 261, 262  
   Theorem: If  $R$  is a Krull domain and  $R/P$  is Noetherian for each height-one prime  $P$ , then  $R$  is Noetherian, 18
- Noether normalization, 372  
 Noetherian flatness  
   Multi-adic completion, 269  
 Noetherian Flatness Theorem, 4, 73, 74, 77, 83, 85, 127, 129, 131, 132, 134, 153, 154, 175, 191, 221, 237, 240, 309, 312, 401, 402  
   for *Homomorphic Image Construction*, 215  
   for *Inclusion Construction*, 74  
 Noetherian limit intersection domain, 76, 127, 186, 225, 226
- Noetherian local ring  
   analytically irreducible, 33  
   analytically normal, 33, 113, 154  
   analytically ramified, 34  
   analytically reducible, 34  
   analytically unramified, 33, 34, 109  
   Cohen-Macaulay, 87, 144  
   depth, 38, 38  
   formal fibers, 40  
   formally equidimensional, 233, 246  
   geometrically normal formal fibers, 231, 244  
   geometrically regular formal fibers, 231, 239, 240, 243  
   quasi-unmixed, 46, 233
- Noetherian ring, 9  
   excellent, 40  
   G-ring, 40  
   J-2, 266
- Noetherian spectrum, 9, 70, 177, 185, 201  
 Non-catenary ring, 173, 175, 176, 186, 197, 198, 209, 211, 221, 234, 244, 404  
 Non-flat locus, 20, 79, 80, 122, 131–134, 175, 176, 186, 189, 190, 198, 201–203, 227  
   of *Insider Construction*, 131  
   defines, 20, 95  
   determines, 20  
   of a homomorphism, 20  
   of a polynomial map, 87, 94, 98, 190, 198  
   of *Insider Construction*, 80, 132
- Non-Noetherian ring, 1, 5, 7, 173, 175, 190, 197, 198, 404  
 Non-smooth locus, 92, 105  
   of a polynomial map, 87, 95, 100  
 Non-uniqueness of representation  
   of power series expressions, 62  
 Nonconstant coefficients, 83

- ideal generated by, 83
- Nor  $R$ , 110
- Normal domain, 140
- Normal fibers, 39
- Normal ideal, 139, 140, 140
- Normal locus, 110
- Normal morphism, 40, 141
- Normal ring, 12, 38, 39, 110, 141
- Northcott, 25
- Ogoma, 2, 4, 6, 210, 232, 245, 246, 252, 258, 261, 262
  - Example: a 3-dimensional Nagata local domain whose generic formal fiber is not equidimensional, 6, 246
  - Example: a normal non-catenary Noetherian local domain, 211, 232, 245, 246
- Ogoma-like example, 251, 255, 405
- Olberding, 7, 180, 183, 206
- Oman, 380
- Order function of an ideal, 13, 22, 343
- P-morphism, 234
- Parameter, regular, 147
- Partial derivative, 92, 100
- PDE extension, 15, 121, 275, 276, 277, 279, 280, 285, 286, 288, 289, 292
- Peskine, 182
- Picavet, 87
- PID, 11, 20, 72
- Popescu, 91
- Power series
  - generalized, 170, 171
- Power series pitfall, 31
- Preservation of excellence
  - under multi-adic completion, 263, 265
  - with Insider Construction, 127, 136
- Preservation of Noetherian
  - under multi-adic completion, 262
- Primarily independent, 6, 277, 280, 283, 285, 287, 293, 295, 298–300, 302, 305, 315
- Primarily limit-intersecting, 4, 310, 310, 312, 315, 318, 324, 325, 327, 327, 330, 332–334, 336, 337
- Prime spectrum, 192
- Prime divisor, 11
- Prime element, 10
- Prime ideal
  - associated to a module, 11, 207, 244
  - dimension of, 10
  - height, 10
  - lost, 178, 179–181, 183
  - structure, 6, 7
  - symbolic power, 11
- Prime spectrum, 9, 36, 173, 190, 200, 202, 375–377, 379, 384, 385
  - diagram, 181, 183, 193, 202, 381, 382
- Principal fractional ideal, 15
- Principal ideal domain, 11
- Properties  $\mathcal{A}$ .1–4
  - relevant for excellence, 104
- Prototype, 52, 54, 55, 131, 209, 217, 221, 223, 225, 228, 240, 402
- Prototype Example
  - Intersection form, 55
- Prototype Theorem, 5, 127, 145, 312
  - Inclusion Version, 129
  - Local Version, 240
  - Local Version, 130, 143, 200
- Pruefer, 205
- Pseudo-geometric ring, 16
- Pullback, 184, 205
- Quadratic transform, local, 49, 165, 166–169, 171, 184, 203, 399
  - iterated, 165
- Quasi-normal algebra, 141
- Quasi-normal morphism, 141
- Quasi-regular sequence, 217
- Quasi-smooth
  - algebra over a ring, 91, 93
- Quasi-unmixed Noetherian local ring, 34, 46, 233
- R-completion, 262
- R-topology, 262
- Radical ideal, 10, 146, 147
- Ratliff, 2, 35, 85, 232, 233, 240, 282
- Ratliff’s Equidimension Theorem, 35, 85, 228, 233, 240, 246
  - a Noetherian local ring  $R$  is universally catenary if and only if the completion of  $R/P$  is equidimensional for every minimal prime  $P$  of  $R$ , 35
- Raynaud, 2, 111, 112
- Reduced fibers, 39
- Reduced ideal, 10
- Reduced ring, 10, 33, 34, 38, 39, 110, 233
- Reduction of an ideal, 139, 140, 140, 142
- Rees, 34, 109
- Rees algebra, 140
- Rees Finite Integral Closure Theorem, 34, 34, 38, 109
  - necessary and sufficient conditions for a Noetherian local ring to be analytically unramified, 34
- Reg, of a ring, 40
- Regular element, 9, 42, 73, 131, 218, 402
- Regular fibers, 39, 39, 88, 89, 340
- Regular for a power series
  - in the sense of Zariski-Samuel, 160
- Regular integral domain, 222
- Regular local domain, 14
- Regular local ring, 2, 5, 7, 14, 33, 38, 40, 47, 88, 110, 130, 165, 167, 168, 170,



- 172, 196, 222, 223, 239, 243, 262, 340, 343, 362, 404
- homomorphic image of, 35, 240
- Regular morphism, 40, 51, 91, 101, 136, 137, 141, 145, 146, 234, 240, 265
- Regular parameter, 147
- Regular sequence, 9, 87, 89, 141, 144, 145, 147, 217, 222, 345, 350, 359
  - on a module, 38
- Regular system of parameters, 14, 196
- Residually algebraically independent, 154, 277, 277, 278, 280, 285–287, 289, 293, 295, 298, 300, 303, 304, 315
- Residually limit-intersecting, 309, 312–315, 318, 320, 322, 324
- Residue field of a ring at a prime ideal, 36
- Riemann Zeta function, 157
- Ring
  - catenary, 34, 231, 232, 234–236, 240, 242, 244
  - coherent regular, 5, 206
  - discrete rank-1 valuation ring, 11
  - equidimensional, 34
  - Henselian, 6, 36, 233, 257, 265
  - Jacobson radical, 11, 61
  - Krull dimension, 10
  - Krull domain, 14
  - local ring, 10
  - Nagata, 16
  - normal ring, 12
  - not universally catenary, 240, 242–244
  - of fractions, 10
  - pseudo-geometric, 16
  - regular local, 2, 5, 7, 14
  - unique factorization domain, 10
  - universally catenary, 34, 231, 232, 234–236, 240, 242–244, 266
  - valuation domain, 11
- Ringed space, 376
- RLR, 14, 14, 33, 35, 39, 138–140, 142–144, 147, 152, 153, 167, 168, 170, 173, 175, 176, 184, 240, 265, 341, 362, 364, 367, 398, 399
- Roitman, 43
- Rond, 76
- Rotthaus, 2–5, 51, 73, 81, 84, 108, 137, 147, 162, 206, 210, 239, 258, 261, 262, 323, 340, 341, 359, 376, 389, 391, 399
- Theorem: dimension of generic formal fiber in excellent rings, 340
  - Example: a 3-dimensional regular local domain that is a Nagata domain and is not excellent, 4, 51, 84, 137
  - Example: a 3-dimensional regular local domain that is a Nagata domain and is not excellent., 3
- Example: A Nagata domain for which the singular locus is not closed, 108, 262
- Theorem: If  $R$  is a Noetherian semilocal ring with geometrically regular formal fibers and  $I_0$  is an ideal of  $R$  contained in the Jacobson radical of  $R$ , then the  $I_0$ -adic completion of  $R$  also has geometrically regular formal fibers, 239
- Rutter, 204, 206
- Salce, 262
- Sally, 1, 46, 181, 323, 340
  - Question: What rings lie between a Noetherian integral domain and its field of fractions?, 1, 46
- Samuel, 15, 39, 160, 386
- Sathaye, 163
- Sather-Wagstaff, 381
- Saydam, 206
- Schanuel, 154
  - conjecture, 158
- Scheme, 104, 375, 376
  - affine, 376
  - formal, 376
  - immersion of, 375, 376, 376, 377
- Schilling, 171
- Schmidt, 1, 2, 49, 113, 141, 374
- Sega, 5, 206
- Seidenberg, 205
- Semilocal ring, 10
- Separated for the  $I$ -adic topology, 87, 88
- Serre, 14, 18
- Serre's conditions, 14
- Seydi, 267
- Shah, 380
- Shannon, 50, 166
- Sharp, 231, 283, 330, 370
- Sheaf, 376
- Sheldon, 395
- Simple PS-extension, 77
- Singular locus of a Nagata domain
  - not closed, 108, 262
- Singular locus of a Noetherian ring, 103, 104, 266
  - closed, 104, 105, 107, 109, 110
- Singularities of algebraic curves, 48
- Smooth
  - 0-smooth, 91
  - algebra over a ring, 91, 253
  - morphism, 91
  - quasi-smooth, 91
- Smoothness
  - relations to the Jacobian ideal, 95
- Spec, 9
- Spec map, 36
- Spectral map, 36, 182, 375, 377–379
- Stable under generalization, 232, 235

- Subspace, *154*
  - topology, *42, 154, 163, 371*
- Swan, 91, 93
- Swanson, 139, 140
- Symbolic power of a prime ideal, *11*
  
- Tanimoto, 91
- Tate, 104, 109
- Taylor, 145
- Tchamna, 262
- Tensor product of modules, 19
- TGF extension, *6, 342, 369, 373, 377–379, 385, 387, 389, 391–400, 405, 406*
- Thompson, 381
- Tight closure, 147
- Topology
  - ideal-adic, *25, 32, 37, 42*
  - linear, *25*
  - subspace, *42, 154, 163, 371*
  - Zariski, 9
- Torsionfree module, *20, 155*
- Total ring of fractions, *10, 401*
- Transcendence degree
  - uncountable, 30
- Trivial generic fiber extension, *6, 7, 342, 378, 391*
  
- UFD, *10, 14, 17, 21, 59, 71, 117, 151, 173, 175, 180, 182, 190, 191, 198, 248–250*
  - Theorem: For  $R$  a UFD,  $x$  a prime element of  $R$  and  $R^*$  the  $x$ -adic completion of  $R$ , the Approximation Domain  $B$  is a UFD, 71
- Unique factorization domain, 5, *10, 71*
- Universality of the construction, 2, 45, *46, 402*
- Universally catenary ring, 5, 7, *34, 40, 41, 85, 104, 110, 176, 186, 211, 217, 224, 231, 232–236, 240, 242–244, 266, 267*
  
- Valabrega, 47, 48, 84, 151, 174, 276, 343, 362, 367
  - consequence of his theorem, 84, 151
  - Theorem:  $C$  a DVR and  $L$  a field with  $C[y] \subset L \subseteq \mathcal{Q}(C[[y]])$  implies  $L \cap C[[y]]$  is an RLR, 47, 151, 317, 325, 343, 362, 367
- Valuation, 12
- Valuation domain, *11, 21, 181, 184, 203*
  - discrete rank-1, 7, *11, 165, 167*
- Value group, 12
- Vamos, 283, 330
- van der Put, 391
  
- Wang, 87
- Weak Flatness Theorem
  - homomorphic image version, 218
  - inclusion version, 118
- Weakly flat, *115, 118–121, 123, 124, 275, 276, 277, 279, 280, 309*
- Weierstrass, 343
- Weierstrass Preparation Theorem, 339, *343, 347*
- Weston, 2, 210, 258, 261, 262
- Wicklein, 381
- Wiegand, R., 31, 73, 184, 205, 283, 330, 370, 380, 383
  - a crucial flatness lemma, 73
- Wiegand, S., 283, 330, 380, 383
  
- Yao, 6, 342, 377, 391
- Yasuda, 378, 389
  - non-TGF extension, 378, 389
  
- Zariski, 39, 49, 106, 109, 112, 160, 166, 167, 182, 386, 400
- Zariski Subspace Theorem, 400
- Zariski topology, *9, 104, 376*
  - closed sets, *9*
  - open sets, *9*
- Zariski's Jacobian criterion
  - for regularity in polynomial rings, 106
- Zariski's Main Theorem, 50, 112, 182
- Zariski-Samuel Commutative Algebra II
  - erroneous theorem, 39
- Zelinsky, 262
- Zero-smooth, *91*
- Zerodivisor, *9*
- Zorn's Lemma, 22