

Preface

This monograph is about *ridge functions*. Recent years have witnessed a flurry of interest in these functions. Ridge functions appear in various fields and under various guises. They appear in fields as diverse as partial differential equations (where they are called *plane waves*), computerized tomography and statistics. These functions are also the underpinnings of many central models in neural networks.

We are interested in ridge functions from the point of view of approximation theory. The basic goal in approximation theory is to approximate complicated objects by simpler objects. Among many classes of multivariate functions, linear combinations of ridge functions are a class of simpler functions. This book studies some problems of approximation of multivariate functions by linear combinations of ridge functions. We present here various properties of these functions. The questions we ask are as follows. When can a multivariate function be expressed as a linear combination of ridge functions from a certain class? When do such linear combinations represent each multivariate function? If a precise representation is not possible, can one approximate arbitrarily well? If well approximation fails, how can one compute/estimate the error of approximation, know that a best approximation exists? How can one characterize and construct best approximations? If a smooth function is a sum of arbitrarily behaved ridge functions, is it true that it can be expressed as a sum of smooth ridge functions? We also study properties of generalized ridge functions, which are very much related to linear superpositions and Kolmogorov's famous superposition theorem. This book ends with a few applications of ridge functions to the problem of approximation by single and two hidden layer neural networks with a restricted set of weights.

I hope that this book will be useful and interesting to both researchers and graduate students.

I would like to express my sincere gratitude to Eriko Hironaka at AMS for handling the review process and for providing professional support. I am also grateful to the Editorial Committee and the anonymous referees for their constructive suggestions which helped to improve the first version of this monograph.