

Index

- $(k + 1)$ -arrow, 76
- 0-cell, 342
- 1-arrow, 50, 65, 67, 68, 72–75, 81, 87, 97, 109, 115, 116, 119, 125, 126, 129, 130, 144, 145, 147, 169, 188, 191, 197, 199, 241, 294
- 2-arrow, 30, 31, 34, 42, 44, 46, 49–52, 55, 62, 63, 65, 66, 69, 71–73, 75, 76, 79, 81, 83, 85, 87, 92, 93, 97, 98, 103, 108–110, 113, 115–117, 119, 120, 126, 127, 129–131, 134–136, 139, 143, 144, 146, 147, 151, 152, 157, 158, 160, 161, 168, 170, 173–176, 182, 183, 188, 191, 192, 197, 199, 215, 222, 225, 226, 241, 253, 294, 355
- 2-dimensional braid, 299
- 2-dimensional braid, simple, 299
- 2-dimensional foam, 63, 122, 158
- 2-morphism, 34, 42
- 3-arrow, 55, 58, 62, 63, 71, 73, 74, 76–82, 84, 85, 87, 92–94, 96–98, 108–110, 113–116, 119, 120, 126, 127, 130, 133–137, 139, 141, 158, 159, 161, 165, 166, 169, 170, 172, 174, 175, 181, 184, 192–194, 197, 199, 215, 222, 225, 226, 239, 241, 246, 249, 250, 253, 259, 261, 295, 330
- 4-arrow, 80, 87, 97, 109, 113, 116, 119, 120, 122, 127, 134, 135, 137, 139, 153, 158, 159, 161–166, 169–171, 181, 184, 187, 197–199, 215, 226–229, 238–249, 251, 253–258, 261, 269, 272, 275, 276, 278, 285, 286, 290, 293, 295, 297, 298, 303–305, 315, 318–320, 322, 334
- 4-simplex, 181, 182, 273
- 5-arrow, 184, 230, 241, 242, 254, 255, 297, 315, 322, 326, 330, 334, 339
- 5-isomorphism, 242, 243, 245, 249, 250, 254–257, 259, 261, 263, 271–273, 276, 279, 286, 290, 295, 297, 307, 322
- 6-arrow, 297, 333, 334
- A -region, 83, 125
- A -sphere, 125
- A -valued cochain, 287
- B -region, 83, 125
- B -sphere, 125
- \mathbb{R}^n , 13, 17, 19, 21, 25
- Θ -sphere, 73
- ψ -move, 197–200, 209, 228, 229, 231, 238, 241, 244, 246, 248, 251–253, 255, 305, 312
- θ -graph, 225
- k -arrow, 76
- k -dimensional disk, 125
- k -disk, 125
- k -handle, 125
- n -cochain, 287
- n -cocycle, 287
- n -cycles, 221, 273
- n -dimensional foam, 189
- n -disk, 125
- n -simplex, 181, 188, 271, 272
- abstract 2-chain, 271
- abstract tensor, 19, 191, 241, 255, 276, 277
- adjoint, 167, 170, 251, 252, 255, 310, 317
- Alexander module, 203
- algebra, 1, 4, 12, 22–28, 32, 34, 39, 45, 49–52, 54, 65, 82, 84, 85, 165, 173, 177, 191, 200, 209, 210, 214–218, 222, 223, 251, 276, 277, 339
- algebra, unit, 23
- anti-symmetric, 183
- arch, 69, 75, 77, 78, 119, 148, 325
- arrow, 8, 11, 12, 14, 17, 29–37, 39–42, 44–52, 55, 58, 62, 63, 65–69, 71–85, 87, 92–94, 96–98, 103, 108–110, 113–117, 119, 120, 122, 125–127, 129–131, 133–137, 139, 141, 143–147, 149–154, 157–166, 168–176, 181–184, 187–189, 191–194, 197–199, 202, 214, 217, 222, 225–231, 238–251, 253–259, 261, 269, 272, 275, 276, 278, 285, 286, 290, 293–295, 297, 298, 303–305, 315, 318–320, 322, 326, 330, 334, 338, 339, 355, 356
- Artin’s braid generators, 214, 294
- associahedron, 2, 181, 184, 260, 279
- associated products, 177
- associative pairing, 23, 25, 50, 63, 218, 253
- attaching region, 83, 125

- attaching sphere, 125
- Banchoff, Thomas, 78
- based binary trees, 177
- basis, 13–20, 22, 23, 25, 27, 173–176, 181, 214, 216, 223, 277–279
- Baton Rouge, 246
- beak-to-beak, 58, 62, 70, 97, 100, 101, 109, 115, 117–119, 122, 127, 133, 145, 146, 149, 154, 158, 170, 171, 242, 250, 255
- belt region, 83, 125
- belt sphere, 125
- bilinear form, 17, 19, 25, 51
- bilinear product, 22–24
- birth, 29, 65, 67, 69, 70, 75–78, 91, 98, 99, 108, 109, 112, 116–120, 123, 125, 129–131, 135, 136, 139, 145–147, 149–151, 153, 158, 169, 170, 218, 241, 242, 251–253, 255, 257, 312
- black vertex, 297, 298, 302, 303, 306, 310–313, 315–318, 320, 321, 323, 325, 337, 338
- blackboard framing, 197
- bookmark, 159, 171, 193, 215, 228–230, 232, 295, 299
- boundary, 42, 64, 65, 72, 73, 83, 97, 110, 111, 113, 125, 158, 168, 170, 176, 177, 181, 182, 184, 185, 187, 188, 194, 203, 204, 221, 238, 271, 273, 274, 276, 284, 285, 290, 293, 299, 302, 311, 326, 330, 333, 336, 338
- boundary operators, 221, 287
- boundary sphere, 125
- braid, 36, 191, 214, 216, 263, 293–295, 298–300, 311, 312, 315, 324, 337, 355
- braid chart, 298, 312, 326, 334
- braid closure, 215, 335
- braid diagram, 294
- braid generator, 295, 296, 330
- braid group, 214, 215, 293–295, 331, 334
- braid index, 296, 299, 301, 306, 313, 323, 330, 333
- braid movie, 299, 302, 315, 330, 331, 334
- braid relations, 214, 294, 295, 297
- braided, 191, 200
- braided hyper-solid, 336, 337
- braided solid, 297, 323, 335
- braided surface, 299
- branch point, 245, 247, 251, 298, 299, 303, 314, 322
- branch value, 298, 303, 311, 322
- branched covering, 298, 299, 303, 311, 312, 314, 322, 323
- Brunnian, 225
- canal, 69, 75, 77, 78, 325
- candy-cane, 251
- Catalan number, 175, 177, 182
- categorify, 4, 90
- category, 8, 12, 29–34, 39, 40, 42, 44, 45, 73, 75, 82, 83, 85, 87, 88, 97, 110, 157, 183, 191, 193, 197, 200, 202, 214, 217, 218, 222, 355
- category, small, 31
- chain complex, 221, 273
- chart, 229, 301
- chart conjugation, 312
- chart movie, 323, 324, 326, 327, 329–334, 336
- chevron move, 312
- Chinese characters, 41
- closed surface braid, 293
- co-core, 125
- co-unit, 51, 53–55, 62–65, 68, 69, 75, 76, 82, 84, 85, 119, 158, 165–167, 169, 170, 218, 222, 223, 251, 297, 325
- coboundary, 287
- cochain, 287
- cocyclic, 287
- codomain, 11
- coloring conditions, 282–284
- conjugation, 202
- coordinate, 13
- core disk, 125
- covering map associated with, 299, 322
- critical cancelation, 97, 98, 119, 120, 127, 135, 143, 145–147, 149, 150, 158, 218, 242, 250, 251, 255
- critical point, 31, 32, 34, 109, 112, 114–116, 120, 122, 123, 152, 175, 193, 194, 197, 198, 200, 218, 229, 231, 251, 302, 323, 338
- crossing, 34, 112, 115, 193–201, 203–206, 209, 213, 215, 216, 228, 231, 233, 246, 250, 272, 276, 288, 293, 294, 302, 303, 318, 319, 325, 338, 356
- curtain, 326
- curtain movie, 333–336
- cuspid, 58, 61, 62, 70, 85, 94–97, 99–103, 105, 108, 109, 112, 114–120, 122, 125–127, 133, 135–138, 140, 145–147, 149, 150, 154, 158, 159, 170, 174, 175, 197–199, 226, 241, 243, 248, 249, 251, 253, 254, 295, 307, 312
- cycle, 221, 273
- death, 29, 65, 67, 69, 70, 75–78, 91, 98, 108, 109, 112, 116–120, 123, 125, 129–131, 135, 136, 139, 145–147, 149–151, 158, 169, 170, 218, 241, 242, 251–253, 255, 257, 312
- decategorify, 90, 96, 106, 116, 141, 147, 159, 182, 197, 200, 215
- degree, 298, 322
- dihedral group, 202

- disk, 83, 125, 203, 204, 217, 218, 298, 299, 311, 312, 324, 327, 328, 330, 331, 333–337
- domain, 11
- double arrow, 30, 31, 34, 42, 44, 49–52, 55, 62, 63, 65, 66, 69, 71–73, 75, 76, 79, 81, 83, 85, 87, 92, 93, 97, 98, 103, 108–110, 113, 115–117, 119, 120, 126, 127, 129–131, 134–136, 139, 143, 144, 147, 151, 152, 157, 158, 160, 161, 168, 170, 173–176, 182, 183, 188, 191, 192, 197, 199, 215, 222, 225, 226, 241, 253, 294, 355
- double curve, 50, 228–233, 235, 236, 242, 243, 248, 257, 284
- dummy variable, 14
- dumpling, 74, 98
- Dyck path, 175
- edge, 64
- Einstein summation convention, 24, 71, 277
- enneahedron, 185
- Equatorial Guinea, 73
- Eskew, Rhea, 159
- exchanger, 30, 34–37, 44, 48, 53, 55, 72, 97, 98, 107–110, 112, 114–116, 123, 135, 146, 159, 161, 167, 170, 171, 177, 184, 185, 193, 195–200, 214, 215, 222, 225, 232, 233, 237–239, 241, 245–251, 253, 255, 261, 290, 295, 303–305, 319, 320, 323, 325, 334, 355
- fish hook, 167, 170, 251, 255, 310, 317
- foam, 54, 62–65, 67, 69, 72, 78–85, 120, 122–124, 158–166, 168–172, 181, 187–189, 229, 231, 238, 241, 253–255, 282, 290, 291, 355
- fold, 52, 53, 57–60, 62, 85, 112–114, 116, 127, 158, 159, 166, 167, 171, 175, 198, 228, 230, 231, 248, 251, 252, 254–256, 314, 337
- fold set, 112
- fold-over map, 175
- fork, 65, 67, 75, 98, 108, 109, 112, 116–120, 123, 125, 132, 135, 136, 140, 145–147, 149–151, 158, 170, 218, 241, 242, 251–253, 255, 257, 312
- framed tangle, 197
- Frobenius algebra, 23, 25–28, 32, 34, 39, 45, 49–52, 54, 65, 82, 84, 85, 165, 216–218
- function, 11–13, 15–17, 19, 23–25, 29, 64, 71, 110–112, 114, 159, 169, 194, 198, 200, 202, 205, 210, 216, 286, 298, 303
- functor, 37, 39, 50, 53, 55, 83, 88, 97, 109, 110, 115, 191, 193, 217, 218
- fundamental group, 203, 205
- fundamental quandle, 203–206
- generating double arrows, 88
- geometric braid, 293
- Globular, 171, 341
- globular composition, 39, 41, 42, 44, 55, 69, 73, 75, 76, 89, 91–93, 97, 113, 158, 192, 193
- globular composition of triple arrows, 73
- globular signature, 349
- glyph, 39, 50, 51, 54, 65, 66, 68, 69, 72, 74, 76–79, 83, 84, 91, 94, 97, 100, 108, 113, 115, 119, 123, 127, 148, 149, 152, 159, 161, 162, 164, 165, 171, 194, 195, 217, 218, 227–238, 241–243, 246, 248, 249, 251, 259, 261, 272, 276, 278, 279, 282, 284, 293, 295–299, 301–303, 312, 314, 318, 326, 330, 355, 356
- group, 202, 205, 214, 221, 223, 241, 271, 274, 277, 287, 330, 331
- group, abelian, 287, 289
- group, braid, 214
- group, dihedral, 202
- group, free abelian, 273, 331
- group, fundamental, 203, 205, 206, 238, 331, 332, 337
- group, knot, 208
- group, permutation, 208, 216, 331, 332
- handcuff, 225
- handlebody, 225, 238, 239, 288
- Hasse diagram, 183, 184
- homeomorphism, 64, 189, 298, 322, 330
- homogeneous triple arrow, 113
- homology, 183, 187, 203, 207, 209, 216, 221, 222, 241, 254, 257, 273, 274, 276, 287
- homomorphism, 202, 203, 205, 221, 222, 273, 287
- homotopy, 203
- Hopf algebra, 216, 222, 223
- horizontal cusp, 70, 97, 103, 105, 109, 115, 118, 119, 124, 128, 136–138, 145, 146, 154, 158, 170, 171, 242, 250, 255
- hyper-solid, 322
- identity braid, 294
- identity function, 11
- independent labels, 283
- index contraction, 13
- initial half-twisted forest, 285
- integral domain, 29
- invariant, 4, 30, 173, 176, 200, 209, 210, 212, 213, 216, 223, 254, 257, 288
- Japanese Society for the Promotion of Science, 9
- Jones polynomial, 4, 173, 177, 191, 209, 213, 216, 218, 222
- Jones, V. F. R., 173, 209
- JSPS, 9
- kanji, 41

- Kauffman bracket, 209
- Kauffman's choice of variable, 173
- Kauffman, Louis, 6, 209
- kernel, 30, 221, 273
- knee, 72
- Kronecker delta, 19, 22, 23, 25, 71
- Kyungpook National University, 8

- left, center, right, 161
- level $(k + 1)$ weak inverses, 76
- lexically ordered, 17
- linear map, 14, 215
- lips, 58, 62, 70, 97, 99–102, 109, 115, 117–119, 122, 127, 133, 145, 146, 149, 154, 158, 170, 171, 242, 250, 255
- local degree, 298
- loop value, 173

- mandu, 74, 98
- manifold, 83, 125, 154, 161, 171, 173, 176, 181, 187–189, 194, 195, 197, 203, 216, 223, 238, 293, 318, 322, 324, 328, 333
- Markov equivalent, 314
- meridional disk, 204
- monoid, 31, 157, 173
- morphism, 8, 31
- Morse function, 194
- movie, 76, 79, 80, 84, 91, 97, 98, 100, 102, 105, 107, 109, 110, 113, 115, 122, 130, 131, 135–137, 139, 149, 154, 258, 298–300, 302, 303, 312, 315
- movie move, 35, 79, 80, 97–100, 102, 105, 107, 110, 115, 130, 131, 149, 154, 248
- movie still, 79, 110, 128, 130, 324, 331, 334, 335
- movie, chart, 323, 324, 326, 327, 329–334, 336
- movie, curtain, 333–336
- multi-category, 30, 39, 45, 73, 87, 88, 97, 106, 109, 110, 116, 118–120, 124–129, 131, 133, 135, 136, 139, 143–145, 150, 157, 169, 182, 183, 188, 191, 193, 197, 198, 200, 229, 241, 242, 253, 293, 294
- multi-functor, 110
- multiple arrow, 119, 120

- natural family of isomorphisms, 37
- naturality, 198, 200, 248, 249, 251, 261, 290, 295, 304, 355
- non-degenerate, 22

- object, 31–34, 41, 42, 44, 45, 52, 65–67, 73, 76, 78, 85, 87, 88, 97, 101, 109, 110, 115, 119, 120, 129, 135, 141, 144–146, 150, 152, 157, 159, 161, 170, 181, 182, 191, 192, 197, 199, 200, 214, 222, 225, 241, 261, 294
- over-arc, 195

- Pachner move, 188
- partial order, 183
- permutation group, 208, 216, 331, 332
- pierogi, 74, 98
- power set, 11
- proper, 111

- quadruple arrow, 80, 87, 97, 109, 113, 116, 119, 120, 122, 127, 134, 135, 137, 139, 153, 158, 159, 161–166, 169–171, 181, 184, 187, 197–199, 215, 226–229, 238–249, 251, 253–258, 261, 269, 272, 275, 276, 278, 285, 286, 290, 293, 295, 297, 298, 303–305, 315, 318–320, 322, 334
- quadruple point, 251, 254
- qualgebra, 255, 269–271, 279, 285, 290
- quandle, 200–202, 204, 205, 215, 241
- quandle cocycle, 205
- quandle homomorphism, 205
- quandle, Alexander, 203, 207, 214
- quandle, dihedral, 202, 203
- quandle, fundamental, 203, 205, 206
- quintuple arrow, 230, 241, 242, 254, 255, 297, 315, 322, 326, 330, 334, 339

- range, 11
- rectangular description, 40
- rectangular diagram, 40
- rectangularly arranged, 17
- reduced product, 175
- reflections, 202
- reflexive, 183
- regular point, 298, 322
- regular value, 298, 322
- Reidemeister, 191, 195–201, 209, 214, 226, 238, 356
- relation, 11
- reverse, 39, 63, 65, 74, 88

- saddle, 65, 67, 70, 75, 98, 108, 109, 112, 116–120, 123, 125, 135, 136, 140, 145–147, 149–151, 158, 170, 218, 241, 242, 251–253, 255, 257, 312
- Saito, Masahico, 259
- scalar multiplication, 13
- seam, 51, 57, 59, 60, 62–64, 82, 85, 158, 159, 162, 163, 165, 230–232, 254–256
- self-distributivity, 201
- self-intersection point, 318
- sesqui-category, 31, 32, 44, 45, 52, 53, 83, 97, 110
- Shakespeare, William, 76, 254
- shalgebra, 241, 255, 257, 269, 270, 273, 274, 277, 278, 282, 283, 286–290
- shark's fin, 50, 158
- simplex, 181, 271
- sink, 318, 356
- skein theory, 210

- skew composition, 39, 41, 42, 44, 70, 73, 74,
 89, 97, 158, 174, 182, 192, 193, 197,
 199, 243, 254, 307
 sliding over optimum, 252
 smooth, 111
 solid, 322
 Solomon Islands, 73
 source, 27, 31, 33, 39–42, 46, 49, 50, 56, 65,
 66, 70–73, 75, 76, 79, 80, 85, 91–95, 97,
 98, 103, 105, 110, 113, 116, 117, 126,
 130, 131, 135, 137, 144, 152–154, 159,
 168, 170, 182, 191, 228, 251, 259, 276,
 279, 285, 286, 318, 322, 342, 356
 sphere, 78, 83, 125, 130, 135, 136, 144, 152,
 169, 170, 238, 311, 312, 314, 315,
 322–330, 332–336
 stabilization, 313
 standard basis, 13
 standard generators, 214, 294
 standard inner product, 19
 Star Trek, 312
 Stasheff polytope, 2, 158, 173, 181,
 184–187, 260, 261, 279
 still, 324
 strongly invertible, 57
 structure constants, 19, 23
 summation convention, 24
 surface, 110
 surface braid, 293, 297–299, 303, 311–316,
 318, 321, 325, 338
 surface braid, closed, 293
 surface with boundary, 110
 swallow-tail, 70, 72–74, 85, 97, 106, 107,
 109, 115, 118, 119, 122, 127, 134, 135,
 142, 145, 146, 154, 158, 170, 171, 242,
 250, 255
 syllepsis, 318–323, 325, 326, 330, 334, 336,
 356
 symmetric group, 208, 216, 331, 332

 tangle, 194
 tangle diagram, 195
 tangle, oriented, 194
 target, 27, 31, 33, 39–42, 46, 50, 56, 65, 66,
 70–73, 75, 76, 79, 80, 85, 91–94, 97, 98,
 103, 105, 110, 113, 116, 117, 126, 130,
 131, 134, 135, 137, 144, 152, 154, 159,
 168, 170, 182, 183, 191, 205, 259, 276,
 279, 286, 322, 342, 356
 Temperley-Lieb algebra, 173, 214, 215
 tensor operator, 18
 tensor product, 17
 tensorand, 18
 tetrahedron, 181

 topological quantum field theory, 217
 TQFT, 217
 transitive, 183
 triangulations, 177, 181, 182, 184–187
 triple arrow, 55, 58, 62, 63, 71, 73, 74,
 76–82, 84, 85, 87, 92–94, 96–98,
 108–110, 113–116, 119, 120, 126, 127,
 130, 133–137, 139, 141, 158, 159, 161,
 165, 166, 169, 170, 172, 174, 175, 181,
 184, 192–194, 197, 199, 215, 222, 225,
 226, 239, 241, 246, 249, 250, 253, 259,
 261, 295, 330
 trivalent graphs, 225–227, 229, 238, 240,
 253, 288, 291
 trivial braid, 294, 299, 311, 324
 type I, 191, 197–201, 209, 212, 226, 228,
 229, 231, 233, 241, 243, 244, 248,
 251–253, 255, 312, 318, 338
 type I bubble, 243
 type I saddle, 243
 type II, 195–198, 200, 201, 209, 210, 226,
 227, 229, 231, 232, 234, 241–244, 248,
 251–253, 295, 303, 305, 307, 310, 312,
 317, 320, 356
 type II bubble, 242
 type II saddle, 242
 type III, 196–198, 201, 209, 211, 214, 226,
 228, 229, 232, 235, 241, 243, 244, 246,
 248, 251–253, 295, 303, 307, 312, 318,
 355

 under-arc, 195, 201, 203, 205, 209
 unit, 12, 23–25, 27, 31, 34, 39, 45–47,
 50–54, 62–65, 68, 69, 75, 76, 82, 84, 85,
 119, 158, 161, 162, 164–166, 169–171,
 218, 223, 251, 297, 325

 vector addition, 13
 vector, coordinate, 13
 vertex of a foam, 63
 vertical composition, 41

 weak inverse, 66
 weak inverses at level $(k + 1)$, 76
 weakly invertible, 63, 66, 146
 white vertex, 302, 303, 307, 310, 311, 318,
 320, 325, 337, 338
 Wirtinger relations, 203
 wonton, 74, 98
 writhe, 209, 213

 Yang-Baxter, 214, 276

 zig-zag, 22, 58, 70, 94, 197–200, 251, 295
 zipper, 67, 68, 75, 81, 82, 120, 122, 166,
 169, 170, 252, 254, 257