

Preface

This book is an English translation of the book [12] with the same title in Japanese published by Saiensu-sha on December 25, 2019. Most parts of chapters 4 and 5 were translated into English in [13] and appeared in the proceedings of IS CRA published by the Iranian Mathematical Society on August 27, 2020.

We remark that the 2-categories dealt with in this book are sometimes called *strict 2-categories*. There is a notion of a *bicategory* that is weaker than the notion of a 2-category. Roughly speaking, a bicategory is defined by replacing the equalities in the axioms of a 2-category by natural isomorphisms. Bicategories have more practical examples¹, but are more complicated to handle than 2-categories. It is known that every bicategory is equivalent to a 2-category as a bicategory (see Leinster [32] for details).

The following are added to this translation. In Appendix A, we added Proposition A.4.4 to give an example of how to treat 2-categories consisting of linear categories. In the Japanese version, due to lack of space, some propositions in Sect. 2.3 were turned into exercises or simply cited without proofs. To make up for this, the translation includes answers to those questions and omitted proofs in Appendix B. Namely, in Sect. B.1, we show that the category of *framed* algebras (see Remark B.1.6) is equivalent to a subcategory of the category of finite linear categories, which gives an answer to Exercise 2.3.9. In Sect. B.2, we give answers to Exercises 2.3.15, 2.3.21, 2.3.23, and 2.3.24 with some explanations in order to turn exercises to propositions. In Sect. B.3, the Krull-Schmidt Theorem (Theorem 2.3.16) is proved using the notion of radical maps. In Sect. B.4, we give two proofs of Theorem 2.3.19, which is a special form of the Morita Theorem. The first proof is elementary; it only requires knowledge in Chapter 2 and a fundamental fact on tensor products over an algebra. The second proof explains a relationship between the first one and the Hom-tensor adjoint. After that we extend this theorem to (one half of) the full version of the Morita theorem by generalizing the second proof. The material in Sect. B.1–B.4 above covers the omitted parts in Sect. 2.3. Sect. B.5 is devoted to the definition of left Kan extensions and its relationships with left adjoint functors, and provides background for the beginning sentences in Sect. 7.1.

¹Such as a bicategory of all small categories whose 1-morphisms are bimodules over small categories, whose compositions are given by tensor products, and whose 2-morphisms are bimodule morphisms between bimodules.

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