Preface

Some decades ago the connections among the areas of inverse problems and sampling were rather tenuous. Researchers in one of the areas were often unfamiliar with the techniques and the capabilities of the other area. Today, the situation, however, has changed, not least because of the intensive study of regularization and sampling methodologies. It turned out that the common thread among these problems led to key constituents in both inverse and sampling problem theory, so that it is time now to pass over to a unifying description of the underlying mathematical concepts. The canonical result is a superordinate framework concerned with the recovery of an object (function, signal, picture) from partial or indirect information about the object.

Recovery problems are characterized by the fact that the transformation from data to object constituents (or vice versa) is a result of the interaction of a certain system with the object that we wish to infer properties about. As a consequence, a good understanding of the system using recovery methods enables the formulation of a mathematical problem by reduction in theoretic sense such that a new, more concrete situation can be efficiently attacked within a well-structured context, remarkably often in reproducing kernel Hilbert and/or Banach spaces. Coherently, by solution processes of recovery problems, the capacity is provided to recognize the causality between the virtuality of the mathematical apparatus and the impact to the scientific reality.

The book allows the reader to concentrate on the essential topics of the *methodologies of recovery problems.* The approach is selective, although a broad palette of recent work including heterogeneous applications is specified in the contents. In order to keep the book manageable, the proofs (available in the literature) are omitted. The special accentuation of the work thus lies in the highlighting of content relationships both intrinsically for inverse and sampling problems as well as in the synopsis of interdependencies. Despite the avoidance of extreme technicalities and elaborate proof techniques, the book requires some basic knowledge in functional analysis, Fourier theory, geometric number theory, constructive approximation, special function theory, etc. For the convenience of the reader, essential cornerstones of these areas are collected as background material (usually without proofs, too). All chapters (including those concerned with the background tools) supply the reader with an adequate overview of relevant references.

The work is a result of the research on recovery problems due to the authors during the last decades. The purpose is to give a consistent overview of scattered results and developments in the field of recovery problems in the form of a monograph. Although the book contains original work, we have tried to make it accessible to students and scientists not only from applied mathematics, but also from all branches of engineering and science. Without being complete a major objective is to provide an addition to the library of any individual interest in recovery theory and to show how advances in this theory lead to new discoveries in other mathematical as well as scientific branches.

It is our great pleasure to thank our colleagues, especially P.L. Butzer, E. Hlawka, C. Müller, G.G. Walter for their specific influence on our scientific thinking. Without their great preliminary achievements, it would not have been possible to complete this book in its present shape.

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