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Basic notation

$\mathbb{Z}, \mathbb{R}, \mathbb{C}$	the sets of integer, real and complex numbers.
\mathbb{R}^d	the d dimensional Euclidean space.
\mathbb{S}^{d-1}	the unit sphere in \mathbb{R}^d .
$\mathbb{P}\mathbb{R}^1$	the real projective space of \mathbb{R}^d .
$G(d, m)$	the Grassmannian manifold of m -dimensional linear subspaces of \mathbb{R}^d .
$\bigwedge^k \mathbb{R}^d$	k -th exterior product of \mathbb{R}^d vector spaces
$(\cdot, \cdot); \langle \cdot, \cdot \rangle$	standard inner product \mathbb{R}^d .
$B(\mathbf{x}, r), \overline{B}_r(\mathbf{x})$	open ball centered at \mathbf{x} of radius r .
$\overline{B}(\mathbf{x}, r), \overline{B}_r(\mathbf{x})$	closed ball centered at \mathbf{x} of radius r .
$B(H, \delta)$	open δ -neighborhood of the set H , that is $B(H, \delta) := \{\mathbf{x} : \exists \mathbf{h} \in H, \ \mathbf{x} - \mathbf{h}\ < \delta\}$.
$\bar{A} = \text{clos}(A)$	closure of the set A .
∂A	boundary of the set A .
$ A $	diameter of the set A .
A^c	complement of the set A .
$\text{int}(A)$	interior of the set A .
$\mathbb{1}_A$	characteristic (indicator) function of the set A .
$\#A$	cardinality of the set A .
$\text{dist}(A, B)$	distance between the sets A, B in a metric space (X, ρ) , that is, $\text{dist}(A, B) := \inf \{\rho(a, b) : a \in A, b \in B\}$.
$\text{dist}_H(A, B)$	Hausdorff distance between the sets A, B in a metric space (X, ρ) , see (1.37).
Σ	symbolic space $\{1, \dots, m\}^{\mathbb{N}}$.
Σ_n	the set $\{1, \dots, m\}^n$.
Σ^*	the set of finite words in the alphabet $\{1, \dots, m\}$, i.e., $\bigcup_{n=0}^{\infty} \Sigma_n$.
σ	the left shift operator on Σ .
$\mathbf{i}, \mathbf{j}, \mathbf{k}, \ell$	elements of $\Sigma^* \cup \Sigma$.
$\mathbf{i} _n$	the finite word formed by the first n symbols of $\mathbf{i} \in \Sigma \cup \Sigma^*$.
\mathbf{i}^-	the finite word obtained by deleting the last symbol of the finite word \mathbf{i} .
$ \mathbf{i} $	length of the finite word $\mathbf{i} \in \Sigma^*$.
$[\mathbf{i}], [i_1 \dots i_n]$	cylinder set on Σ , i.e. $\{\mathbf{j} \in \Sigma : j_k = i_k \text{ for } k = 1, \dots, n\}$.
Π	the natural projection from the symbolic space Σ to \mathbb{R}^d , see (1.42).

$\mathbf{i} \wedge \mathbf{j}$	the common prefix of \mathbf{i} and \mathbf{j} for $\mathbf{i}, \mathbf{j} \in \bigcup_{n=1}^{\infty} \{1, \dots, m\}^n \cup \{1, \dots, m\}^{\mathbb{N}}$.
$S_{i_1 \dots i_n}, S_{\mathbf{i}}$	n -fold composition: $S_{i_1}(S_{i_2}(\dots(S_{i_n}(x)))) = S_{i_1} \circ \dots \circ S_{i_n}(x)$ corresponding to the finite word $\mathbf{i} = (i_1, \dots, i_n)$.
$\dim_S(\Lambda), \dim_S(\nu)$	similarity dimension of the self-similar set and measure.
$s(A_1, \dots, A_m), \dim_{\text{aff}}(\mathcal{F})$	affinity dimension of the self-affine IFS $\mathcal{F} = (A_i x + t_i)_{i=1}^m$
$r_{\mathbf{i}}$	the product $r_{i_1} \dots r_{i_n}$ for a finite word $\mathbf{i} = (i_1 \dots i_n)$.
$p_{\mathbf{i}}$	the product of probabilities $p_{i_1} \dots p_{i_n}$ for a finite word $\mathbf{i} = (i_1 \dots i_n)$.
T^n	for a map $T : X \rightarrow X$ we write T^n for the n -fold composition of T , that is, $T^n := \underbrace{T \circ \dots \circ T}_n$.
$\Lambda_{i_1 \dots i_n}$ $\widetilde{\mathcal{M}}_r, \mathcal{M}_r$	geometric cylinder set $S_{i_1 \dots i_n}(\Lambda) = \Pi([i_1 \dots i_n])$. symbolic Moran minimal cut-set $\{\mathbf{i} \in \Sigma : \Lambda_{\mathbf{i}} \leq r < \Lambda_{\mathbf{i}^-} \}$ at scale r , and the geometric Moran cover $\{\Pi([\mathbf{i}])\}_{\mathbf{i} \in \widetilde{\mathcal{M}}_r}$.
Proj_{θ}	orthogonal projection along the line in the direction $\theta \in \mathbb{P}\mathbb{R}^{d-1}$.
$\text{proj}_{\mathbf{v}}$	orthogonal projection onto the line determined by the vector \mathbf{v} .
π_i	projection to the i -th axis in \mathbb{R}^d for $d \geq 2$, that is, $\pi_i(x_1, \dots, x_d) := x_i$ for $(x_1, \dots, x_d) \in \mathbb{R}^d$.
$\mu _A$ $\mu_{\xi}^{\mathbf{x}}, \mu_{\mathbf{x}}^{\mathbf{v}}$	restriction of the measure μ to the set A . the family of conditional measures of μ with respect to the measurable partition ξ and, in particular, for the partition defined by $\text{proj}_{\mathbf{v}}^{-1}$.
$f_*\mu$	push-forward of the measure μ with respect to f , that is, $f_*\mu(A) = \mu(f^{-1}(A))$.
\mathcal{L}^d	Lebesgue measure on \mathbb{R}^d .
\mathcal{H}^s	s -dimensional Hausdorff measure.
\mathcal{H}_{∞}^s	s -dimensional Hausdorff content.
$\mathcal{P}^s, \widetilde{\mathcal{P}}^s$	s -dimensional packing measure and s -dimensional outer packing measure.
$\dim_{\text{H}}(A)$	Hausdorff dimension of the set A .
$\dim_{\text{P}}(A)$	packing dimension of the set A .
$N_{\delta}(A)$	the smallest number of sets of diameter δ needed to cover the bounded set A .
$\overline{\dim}_{\text{B}}(A), \underline{\dim}_{\text{B}}(A)$	upper and lower box dimensions of the set A ; if it exists it is denoted by $\dim_{\text{B}}(A)$.
$\dim_{\text{A}}(A)$	Assouad dimension of the set A .
$\dim_{\text{F}}(\mu), \dim_{\text{F}}(A)$	Fourier dimension of the measure μ and the set A .
$\widehat{f}, \widehat{\mu}$	Fourier transform of the function f and the measure μ .

$\mu \ll \nu$	the measure μ is absolutely continuous with respect to ν .
$\mu \perp \nu$	the probability measures μ and ν are mutually singular.
$L^q(\mathbb{R}^d)$	space of (equivalence classes of) functions f for which $\int f ^q d\mathcal{L}^d < \infty$.
$\mathcal{E}_t(\mu)$	t -energy of the measure μ .
$\text{spt}(\mu)$	support of the measure μ .
$\underline{D}(\nu, x)$	lower derivative of the measure ν at x .
$\frac{d\nu}{dx}$	Radon-Nikodym derivative of the measure ν with respect to the Lebesgue measure.
$\Theta_*^s(\nu, x), \Theta^{*s}(\nu, x)$	lower and upper s -densities of the measure ν at x .
$\overline{d}_\mu(x), \underline{d}_\mu(x)$	upper and lower local dimension of the measure μ at x ; if it exists it is denoted by $d_\mu(x)$.
$\chi_{\mathbb{F}}^{\mathbf{P}}, \chi_\mu$	Lyapunov exponent of the measure $\mathbf{p}^{\mathbb{N}} = (p_1, \dots, p_m)^{\mathbb{N}}$ and of the measure μ in self-similar case.
$\chi_i, \chi_i(\mu)$	The i th Lyapunov exponent of the measure μ in self-affine case.
$h_{\mathbf{p}}$	entropy of the Bernoulli measure $\mathbf{p}^{\mathbb{N}}$
$\overline{D}_q(\mu)$	L^q dimension of the measure μ .
$\overline{\dim}_e(\mu), \underline{\dim}_e(\mu)$	upper and lower entropy dimension of the measure μ .
$\overline{\dim}_H(\mu), \underline{\dim}_H(\mu)$	upper and lower Hausdorff dimension of the measure μ .
$\tau(\mu, q)$	L^q spectrum of the measure μ .
\mathcal{D}_n and \mathcal{D}_n	partition of \mathbb{R}^d into dyadic cubes.
$H_\mu(\mathcal{P}), H(\mu, \mathcal{P})$	entropy of the partition \mathcal{P} with respect to μ .
$I_\mu(\xi \mathcal{B}), H_\mu(\xi \mathcal{B})$	conditional information and conditional entropy of the countable partition ξ given a partition or sub- σ -algebra \mathcal{B} .
$h_n(\mu)$	entropy of the partition \mathcal{D}_n .
$H_r(\mu)$	r -scale entropy of μ .
$\ \nu\ _{2,\gamma}$	homogeneous Sobolev γ -norm: $\ \nu\ _{2,\gamma}^2 = \int_{\mathbb{R}^n} \widehat{\nu}(\xi) ^2 \xi ^{2\gamma} d\xi$.
$M_d(\mathbb{R}), M_d(\mathbb{Z})$	the set of $d \times d$ matrices with elements from \mathbb{R} and \mathbb{Z} .
$GL(d, \mathbb{R})$	the set of invertible $d \times d$ matrices with elements from \mathbb{R} .
$SL(d, \mathbb{R})$	the set of invertible $d \times d$ matrices with elements from \mathbb{R} of determinant 1.
$O(d, \mathbb{R}), SO(d, \mathbb{R})$	orthogonal subgroup and special orthogonal subgroup.
A^*	transpose of the matrix A .
$\alpha_i(A)$	i th singular value of the matrix A .
$\varphi^s(A)$	singular value function $\varphi^s(A) = \alpha_1(A) \cdots \alpha_{\lfloor s \rfloor}(A)^{s - (\lfloor s \rfloor - 1)}$.
$A^{\wedge k}$	k th exterior product of the matrix A .
$A _V$	linear operator of A restricted to the subspace V .

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