Index

Basic notation

$\mathbb{Z}, \mathbb{R}, \mathbb{C}$	the sets of integer, real and complex numbers.
$\mathbb{R}^d$	the d dimensional Euclidean space.
$\mathbb{S}^{d-1}$	the unit sphere in $\mathbb{R}^d$.
$\mathbb{PR}^1$	the real projective space of $\mathbb{R}^d$.
$G(d,m)$	the Grassmannian manifold of m-dimensional linear subspaces of $\mathbb{R}^d$.
$\bigwedge^k \mathbb{R}^d$	k-th exterior product of $\mathbb{R}^d$ vector spaces
$(\cdot,\cdot); \langle\cdot,\cdot\rangle$	standard inner product $\mathbb{R}^d$.
$B(\mathbf{x},r), B_r(\mathbf{x})$	open ball centered at $\mathbf{x}$ of radius r.
$\overline{B}(\mathbf{x},r), \overline{B}_r(\mathbf{x})$	closed ball centered at $\mathbf{x}$ of radius r.
$B(H,\delta)$	open δ-neighborhood of the set H, that is $B(H,\delta) := \{\mathbf{x} : \exists \mathbf{h} \in H,\ \|\mathbf{x}-\mathbf{h}\| < \delta\}$.
$\bar{A} = \mathrm{clos}(A)$	closure of the set A.
∂A	boundary of the set A.
$\|A\|$	diameter of the set A.
A^c	complement of the set A.
$\mathrm{int}(A)$	interior of the set A.
$\mathbb{1}_A$	characteristic (indicator) function of the set A.
$\#A$	cardinality of the set A.
$\mathrm{dist}(A,B)$	distance between the sets A,B in a metric space (X,ρ), that is, $\mathrm{dist}(A,B) := \inf\{\rho(a,b) : a \in A, b \in B\}$.
$\mathrm{dist}_H(A,B)$	Hausdorff distance between the sets A,B in a metric space (X,ρ), see (1.37).
Σ	symbolic space $\{1,\dots,m\}^{\mathbb{N}}$.
Σ_n	the set $\{1,\dots,m\}^n$.
Σ^*	the set of finite words in the alphabet $\{1,\dots,m\}$, i.e., $\bigcup_{n=0}^{\infty} \Sigma_n$.
σ	the left shift operator on Σ.
$\mathbf{i},\mathbf{j},\mathbf{k},\boldsymbol{\ell}$	elements of $\Sigma^* \cup \Sigma$.
$\mathbf{i}\|_n$	the finite word formed by the first n symbols of $\mathbf{i} \in \Sigma \cup \Sigma^*$.
$\mathbf{i}^-$	the finite word obtained by deleting the last symbol of the finite word $\mathbf{i}$.
$\|\mathbf{i}\|$	length of the finite word $\mathbf{i} \in \Sigma^*$.
$[\mathbf{i}], [i_1 \dots i_n]$	cylinder set on Σ, i.e. $\{\mathbf{j} \in \Sigma : j_k = i_k \text{ for } k = 1,\dots,n\}$.
Π	the natural projection from the symbolic space Σ to $\mathbb{R}^d$, see (1.42).

$\mathbf{i} \wedge \mathbf{j}$	the common prefix of $\mathbf{i}$ and $\mathbf{j}$ for $\mathbf{i}, \mathbf{j} \in \bigcup_{n=1}^{\infty} \{1, \ldots, m\}^n \cup \{1, \ldots, m\}^{\mathbb{N}}$.
$S_{i_1 \ldots i_n}, S_{\mathbf{i}}$	n-fold composition: $S_{i_1}(S_{i_2}(\cdots(S_{i_n}(x)))) = S_{i_1} \circ \cdots \circ S_{i_n}(x)$ corresponding to the finite word $\mathbf{i} = (i_1, \ldots, i_n)$.
$\dim_{\mathrm{S}}(\Lambda), \dim_{\mathrm{S}}(\nu)$	similarity dimension of the self-similar set and measure.
$s(A_1, \ldots, A_m)$, $\dim_{\mathrm{aff}}(\mathcal{F})$	affinity dimension of the self-affine IFS $\mathcal{F} = (A_i x + t_i)_{i=1}^m$
$r_{\mathbf{i}}$	the product $r_{i_1} \cdots r_{i_n}$ for a finite word $\mathbf{i} = (i_1 \ldots i_n)$.
$p_{\mathbf{i}}$	the product of probabilities $p_{i_1} \cdots p_{i_n}$ for a finite word $\mathbf{i} = (i_1 \ldots i_n)$.
T^n	for a map $T : X \to X$ we write T^n for the n-fold composition of T, that is, $T^n := \underbrace{T \circ \cdots \circ T}_{n}$.
$\Lambda_{i_1 \ldots i_n}$	geometric cylinder set $S_{i_1 \ldots i_n}(\Lambda) = \Pi([i_1 \ldots i_n])$.
$\widetilde{\mathcal{M}}_r, \mathcal{M}_r$	symbolic Moran minimal cut-set $\{\mathbf{i} \in \Sigma : \|\Lambda_{\mathbf{i}}\| \leqslant r < \|\Lambda_{\mathbf{i}^-}\|\}$ at scale r , and the geometric Moran cover $\{\Pi([\mathbf{i}])\}_{\mathbf{i} \in \widetilde{\mathcal{M}}_r}$
Proj_θ	orthogonal projection along the line in the direction $\theta \in \mathbb{PR}^{d-1}$.
$\mathrm{proj}_{\mathbf{v}}$	orthogonal projection onto the line determined by the vector $\mathbf{v}$.
π_i	projection to the i-th axis in $\mathbb{R}^d$ for $d \geqslant 2$, that is, $\pi_i(x_1, \ldots, x_d) := x_i$ for $(x_1, \ldots, x_d) \in \mathbb{R}^d$.
$\mu\|_A$	restriction of the measure μ to the set A.
$\mu_x^\xi, \mu_x^{\mathbf{v}}$	the family of conditional measures of μ with respect to the measurable partition ξ and, in particular, for the partition defined by $\mathrm{proj}_{\mathbf{v}}^{-1}$.
$f_*\mu$	push-forward of the measure μ with respect to f, that is, $f_*\mu(A) = \mu(f^{-1}(A))$.
$\mathcal{L}^d$	Lebesgue measure on $\mathbb{R}^d$.
$\mathcal{H}^s$	s-dimensional Hausdorff measure.
$\mathcal{H}_\infty^s$	s-dimensional Hausdorff content.
$\mathcal{P}^s, \widetilde{\mathcal{P}}^s$	s-dimensional packing measure and s-dimensional outer packing measure.
$\dim_{\mathrm{H}}(A)$	Hausdorff dimension of the set A.
$\dim_{\mathrm{P}}(A)$	packing dimension of the set A.
$N_\delta(A)$	the smallest number of sets of diameter δ needed to cover the bounded set A.
$\overline{\dim}_{\mathrm{B}}(A)$, $\underline{\dim}_{\mathrm{B}}(A)$	upper and lower box dimensions of the set A; if it exists it is denoted by $\dim_{\mathrm{B}}(A)$.
$\dim_{\mathrm{A}}(A)$	Assouad dimension of the set A.
$\dim_{\mathrm{F}}(\mu), \dim_{\mathrm{F}}(A)$	Fourier dimension of the measure μ and the set A.
$\widehat{f}, \widehat{\mu}$	Fourier transform of the function f and the measure μ.

$\mu \ll \nu$	the measure μ is absolutely continuous with respect to ν.
$\mu \perp \nu$	the probability measures μ and ν are mutually singular.
$L^q(\mathbb{R}^d)$	space of (equivalence classes of) functions f for which $\int \lvert f\rvert^q d\mathcal{L}^d < \infty$.
$\mathcal{E}_t(\mu)$	t-energy of the measure μ.
$\mathrm{spt}(\mu)$	support of the measure μ.
$\underline{D}(\nu, x)$	lower derivative of the measure ν at x.
$\frac{d\nu}{dx}$	Radon-Nikodym derivative of the measure ν with respect to the Lebesgue measure.
$\Theta^s_*(\nu, x), \Theta^{*s}(\nu, x)$	lower and upper s-densities of the measure ν at x.
$\overline{d}_\mu(x), \underline{d}_\mu(x)$	upper and lower local dimension of the measure μ at x; if it exists it is denoted by $d_\mu(x)$.
$\chi^{\mathbf{p}}_{\mathbf{r}}, \chi_\mu$	Lyapunov exponent of the measure $\mathbf{p}^{\mathbb{N}} = (p_1, \dots, p_m)^{\mathbb{N}}$ and of the measure μ in self-similar case.
$\chi_i, \chi_i(\mu)$	The ith Lyapunov exponent of the measure μ in self-affine case.
$h_{\mathbf{p}}$	entropy of the Bernoulli measure $\mathbf{p}^{\mathbb{N}}$
$\underline{D_q}(\mu)$	L^q dimension of the measure μ.
$\overline{\dim}_e(\mu), \underline{\dim}_e(\mu)$	upper and lower entropy dimension of the measure μ.
$\overline{\dim}_{\mathrm{H}}(\mu), \underline{\dim}_{\mathrm{H}}(\mu)$	upper and lower Hausdorff dimension of the measure μ.
$\tau(\mu, q)$	L^q spectrum of the measure μ.
$\mathcal{D}_n$ and $\mathscr{D}_n$	partition of $\mathbb{R}^d$ into dyadic cubes.
$H_\mu(\mathcal{P}), H(\mu, \mathcal{P})$	entropy of the partition $\mathcal{P}$ with respect to μ.
$I_\mu(\xi \vert \mathcal{B}), H_\mu(\xi \vert \mathcal{B})$	conditional information and conditional entropy of the countable partition ξ given a partition or sub-σ-algebra $\mathcal{B}$.
$h_n(\mu)$	entropy of the partition $\mathcal{D}_n$.
$H_r(\mu)$	r-scale entropy of μ.
$\lVert\nu\rVert_{2,\gamma}$	homogeneous Sobolev γ-norm: $\lVert\nu\rVert^2_{2,\gamma} = \int_{\mathbb{R}^n} \lvert\widehat{\nu}(\xi)\rvert^2 \lvert\xi\rvert^{2\gamma} \, d\xi$.
$M_d(\mathbb{R}), M_d(\mathbb{Z})$	the set of $d \times d$ matrices with elements from $\mathbb{R}$ and $\mathbb{Z}$.
$GL(d, \mathbb{R})$	the set of invertible $d \times d$ matrices with elements from $\mathbb{R}$.
$SL(d, \mathbb{R})$	the set of invertible $d \times d$ matrices with elements from $\mathbb{R}$ of determinant 1.
$O(d, \mathbb{R}), SO(d, \mathbb{R})$	orthogonal subgroup and special orthogonal subgroup.
A^*	transpose of the matrix A.
$\alpha_i(A)$	ith singular value of the matrix A.
$\varphi^s(A)$	singular value function $\varphi^s(A) = \alpha_1(A) \cdots \alpha_{\lceil s\rceil}(A)^{s-(\lceil s\rceil-1)}$.
$A^{\wedge k}$	kth exterior product of the matrix A.
$A\vert_V$	linear operator of A restricted to the subspace V.

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