

Contents

This is a three-volume work; each volume consists of two parts. The preface and backmatter are identical across all three volumes. The table of contents for each volume is followed by a copy of the tables of contents for the other two volumes.

Reference Convention. In each of Volumes II and III, the numbering of parts, chapters, and pages restarts at 1. Page numbers are prefixed with a volume number so that, for example, page n of Volume II is printed II. n . References (chapters, definitions, theorems, equations, etc.) to a different volume are preceded by I, II, or III according to the volume. For example, $a.b.c$ in Volume I is denoted by I. $a.b.c$ in Volumes II and III, and Chapter N in Volume III is denoted by Chapter III. N in Volumes I and II.

Preface	xv
Bimonoidal and E_n -Monoidal Categories	xv
Applications in Quantum Science	xvi
Applications in Algebraic K -Theory	xvii
Audience and Features	xviii
Part and Chapter Summaries	xx
Reading Guides	xxx
Part 1. Enriched Monoidal Categories and Multicategories	III.1
Chapter 1. Enriched Monoidal Categories	III.3
1.1. Review of Monoidal Categories	III.3
1.2. Enriched Categories, Functors, and Natural Transformations	III.13
1.3. The Tensor Product of Enriched Categories	III.21
1.4. Monoidal Enriched Categories	III.37
1.5. Cat-Monoidal 2-Categories	III.53
1.6. Notes	III.54
Chapter 2. Change of Enrichment	III.57
2.1. Change of Enriching Categories	III.57
2.2. 2-Functoriality of Change of Enrichment	III.62
2.3. Preservation of Enriched Tensor	III.68
2.4. Preservation of Enriched Monoidal Structure	III.74
2.5. Coherence of Enriched Monoidal Categories	III.83
2.6. Strictification of Enriched Monoidal Categories	III.86
2.7. Notes	III.90

Chapter 3. Self-Enrichment and Enriched Yoneda	III.91
3.1. Self-Enriched Categories	III.91
3.2. Represented Enriched Functors	III.95
3.3. Self-Enriched Symmetric Monoidal Categories	III.103
3.4. Enriched Yoneda Bijection	III.108
3.5. Enriched Ends and Internal Mapping Objects	III.113
3.6. Enriched Yoneda Lemma	III.125
3.7. Symmetric Monoidal Diagram Categories	III.132
3.8. Enriched Diagram Categories	III.142
3.9. Tensor and Cotentor Enriched Categories	III.147
3.10. Notes	III.155
Chapter 4. Pointed Objects, Smash Products, and Pointed Homs	III.157
4.1. Smash Products	III.158
4.2. Pointed Homs	III.165
4.3. Pointed Diagram Categories	III.167
4.4. Notes	III.175
Chapter 5. Multicategories	III.177
5.1. The 2-Category of Multicategories	III.177
5.2. The Cartesian Structure on Multicategories	III.184
5.3. Permutative Categories as Pointed Multicategories	III.186
5.4. Limits and Colimits of Monadic Algebras	III.190
5.5. Limits and Colimits of Multicategories	III.196
5.6. Tensor and Smash Products of Multicategories	III.201
5.7. The Internal Hom for Multicategories	III.207
5.8. Notes	III.219
Chapter 6. Enriched Multicategories	III.221
6.1. Enriched Multicategories	III.222
6.2. Change of Enriching Categories	III.228
6.3. Enriched Endomorphism Multicategories	III.230
6.4. The Multicategory of Small Multicategories	III.234
6.5. Permutative Categories and Multilinearity	III.237
6.6. The Multicategory of Small Permutative Categories	III.241
6.7. Notes	III.250
Part 2. Algebraic K-Theory	III.251
Chapter 7. Homotopy Theory Background	III.253
7.1. Simplicial Objects	III.254
7.2. Simplicial Homotopy and Nerve	III.259
7.3. Symmetric Sequences of Pointed Simplicial Sets	III.262
7.4. Symmetric Spectra	III.265
7.5. Limits and Colimits of Symmetric Spectra	III.269
7.6. Smash Products, Internal Hom, and (Co)tensor Structure of Symmetric Spectra	III.271
7.7. Quillen Model Categories	III.277
7.8. Examples of Quillen Model Categories	III.284
7.9. Notes	III.286

Chapter 8. Segal K -Theory of Permutative Categories	III.289
8.1. Categories of Γ -Objects	III.290
8.2. Symmetric Spectra from Γ -Simplicial Sets	III.294
8.3. Γ -Categories from Permutative Categories	III.296
8.4. Partition Multicategories	III.303
8.5. Segal J -Theory and K -Theory	III.309
8.6. Notes	III.314
Chapter 9. Categories of \mathcal{G}_* -Objects	III.317
9.1. The Category \mathcal{G}	III.318
9.2. Symmetric Monoidal Closed Structure for \mathcal{G}_* -Objects	III.326
9.3. Symmetric Spectra from \mathcal{G}_* -Simplicial Sets	III.331
9.4. $\mathbf{K}^{\mathcal{G}}$ is Symmetric Monoidal	III.335
9.5. Notes	III.345
Chapter 10. Elmendorf-Mandell K -Theory of Permutative Categories	III.347
10.1. The Partition Product	III.351
10.2. Characterization of $\mathcal{M}\underline{1}$ -Modules	III.362
10.3. Elmendorf-Mandell J -Theory and K -Theory	III.370
10.4. Elmendorf-Mandell \mathcal{G}_* -categories	III.380
10.5. An Equivalent Description of Elmendorf-Mandell J -theory	III.386
10.6. Equivalence Between Segal K -Theory and Elmendorf-Mandell K -Theory	III.392
10.7. (Co)lax and Strong Elmendorf-Mandell \mathcal{G}_* -Categories	III.397
10.8. Notes	III.408
Chapter 11. K -Theory of Ring and Bipermutative Categories	III.413
11.1. The Associative Operad	III.415
11.2. Detecting Ring Categories	III.420
11.3. K -Theory Spectra of Ring Categories	III.425
11.4. The Barratt-Eccles Operad	III.431
11.5. Detecting Bipermutative Categories	III.439
11.6. K -Theory Spectra of Bipermutative Categories	III.441
11.7. Notes	III.447
Chapter 12. K -Theory of Braided Ring Categories	III.449
12.1. The Braid Operad	III.450
12.2. The Braid Operad is an E_2 -Operad	III.457
12.3. Coherence of the Braid Operad	III.462
12.4. Detecting Braided Ring Categories	III.468
12.5. K -Theory Spectra of Braided Ring Categories	III.471
12.6. Notes	III.473
Chapter 13. K -Theory of E_n -Monoidal Categories	III.475
13.1. The Iterated Monoidal Category Operad	III.477
13.2. The Iterated Monoidal Category Operad is an E_n -Operad	III.485
13.3. Coherence of the Iterated Monoidal Category Operad	III.490
13.4. Detecting E_n -Monoidal Categories	III.496
13.5. K -Theory Spectra of E_n -Monoidal Categories	III.499
13.6. Notes	III.501

Bibliography and Indices	III.503
Appendix A. Open Questions	III.505
A.1. Bimonoidal Categories	III.505
A.2. E_n -Monoidal Categories	III.508
A.3. Enriched Monoidal Categories	III.512
A.4. Homotopy Theory	III.513
A.5. Algebraic K -Theory	III.514
Bibliography	III.517
List of Main Facts	III.529
List of Notations	III.549
Index	III.565

Main Contents of Volume I

Part 1. Symmetric Bimonoidal Categories	I.1
Chapter 1. Basic Category Theory	I.3
1.1. Categories	I.3
1.2. Monoidal Categories	I.10
1.3. Coherence	I.15
1.4. Notes	I.18
Chapter 2. Symmetric Bimonoidal Categories	I.19
2.1. Definitions	I.20
2.2. Reduction of Axioms	I.26
2.3. Distributive Symmetric Monoidal Categories	I.32
2.4. Finite Ordinal Category	I.33
2.5. Bipermutative Categories	I.40
2.6. Application: Reversible Programming of Finite Types	I.45
2.7. Notes	I.48
Chapter 3. Coherence of Symmetric Bimonoidal Categories	I.51
3.1. Regularity	I.53
3.2. Induction Devices	I.61
3.3. Reduction of Additive and Multiplicative Zeros	I.67
3.4. Zero Reduction of Paths	I.73
3.5. Existence of Zero Reduction of Paths	I.79
3.6. Reduction of Distributivity	I.94
3.7. Zero and Delta Reduction of Paths	I.107
3.8. Reduction of Multiplicative Units	I.118
3.9. The First Coherence Theorem	I.123
3.10. Coherence of Bimonoidal Categories	I.128
3.11. Notes	I.131
Chapter 4. Coherence of Symmetric Bimonoidal Categories II	I.135
4.1. Motivation	I.137

4.2. The Distortion Category	I.139
4.3. The Distortion of a Path	I.151
4.4. The Second Coherence Theorem	I.158
4.5. Coherence of Bimonoidal Categories II	I.161
4.6. Distortion Categories as Grothendieck Constructions	I.164
4.7. Notes	I.167
Chapter 5. Strictification of Tight Symmetric Bimonoidal Categories	I.171
5.1. Symmetric Bimonoidal Functors	I.173
5.2. Associated Right Bipermutative Category: Definitions	I.180
5.3. Associated Right Bipermutative Category: Proofs	I.188
5.4. Strictification	I.193
5.5. Strictification of Tight Bimonoidal Categories	I.196
5.6. Notes	I.203
Part 2. Bicategorical Aspects of Symmetric Bimonoidal Categories	I.205
Chapter 6. Definitions from Bicategory Theory	I.207
6.1. Bicatagories and 2-Categories	I.209
6.2. Lax Functors, Lax Transformations, and Pastings	I.214
6.3. Modifications and Adjunctions	I.221
6.4. Monoidal Bicategories	I.224
6.5. Symmetric Monoidal Bicategories	I.229
6.6. The Gray Tensor Product	I.239
6.7. Permutative Gray Monoids and 2-Categories	I.246
Chapter 7. Baez's Conjecture	I.255
7.1. The 2-Category of Symmetric Bimonoidal Categories	I.259
7.2. The Additive Structure	I.262
7.3. The Multiplicative Structure	I.264
7.4. Weakly Initial Symmetric Bimonoidal Category	I.276
7.5. Coherence of Symmetric Bimonoidal Functors	I.279
7.6. Uniqueness of 2-Cells	I.284
7.7. Initial 1-Cell	I.285
7.8. Bi-Initial Symmetric Bimonoidal Category	I.291
7.9. Notes	I.292
Chapter 8. Symmetric Monoidal Bicategorification	I.295
8.1. Matrix Construction	I.301
8.2. The Base Unitors	I.306
8.3. The Base Associator	I.310
8.4. The Matrix Bicategory	I.314
8.5. The Monoidal Identity	I.325
8.6. The Monoidal Composition	I.327
8.7. The Pseudofunctoriality of the Monoidal Composition	I.334
8.8. The Monoidal Associator	I.352
8.9. The Monoidal Unitors	I.376
8.10. The Pentagonator	I.385
8.11. The 2-Unitors	I.393

8.12.	The Matrix Monoidal Bicategory	I.398
8.13.	The Braiding	I.401
8.14.	The Matrix Braided Monoidal Bicategory	I.412
8.15.	The Matrix Symmetric Monoidal Bicategory	I.421

Main Contents of Volume II

Part 1.	Braided Bimonoidal Categories	II.1
Chapter 1.	Preliminaries on Braided Structures	II.3
1.1.	The Braid Groups	II.4
1.2.	Elementary Block Braids	II.9
1.3.	Braided Monoidal Categories	II.13
1.4.	The Drinfeld Center	II.21
1.5.	The Symmetric Center	II.31
1.6.	Coherence of Braided Monoidal Categories	II.32
1.7.	Notes	II.34
Chapter 2.	Braided Bimonoidal Categories	II.35
2.1.	Definitions	II.36
2.2.	Recovering Laplaza's Axioms	II.44
2.3.	Abelian Categories	II.46
2.4.	Abelian Categories with a Braiding	II.54
2.5.	Abelian Categories with a Monoidal Structure	II.61
2.6.	Notes	II.62
Chapter 3.	Applications to Quantum Groups and Topological Quantum Computation	II.65
3.1.	Braided Bialgebras	II.67
3.2.	Modules over Braided Bialgebras	II.75
3.3.	Fibonacci Anyons: The Monoidal Structure	II.81
3.4.	Fibonacci Anyons: The Braided Bimonoidal Structure	II.87
3.5.	Ising Anyons: The Monoidal Structure	II.90
3.6.	Ising Anyons: The Braided Bimonoidal Structure	II.102
3.7.	Notes	II.106
Chapter 4.	Bimonoidal Centers	II.109
4.1.	The Bimonoidal Drinfeld Center: Definition	II.111
4.2.	The Additive Structure	II.116
4.3.	The Multiplicative Structure	II.121
4.4.	The Multiplicative Zeros and Distributivity	II.122
4.5.	The Bimonoidal Symmetric Center	II.124
Chapter 5.	Coherence of Braided Bimonoidal Categories	II.127
5.1.	Permutative Braided Categories	II.130
5.2.	The Braided Distortion Category	II.134
5.3.	The Braided Distortion of a Path	II.144
5.4.	The Coherence Theorem	II.149
5.5.	Braided Distortion as a Grothendieck Construction	II.153

Chapter 6. Strictification of Tight Braided Bimonoidal Categories	II.157
6.1. Braided Bimonoidal Functors	II.160
6.2. Associated Right Permutative Braided Category	II.164
6.3. Strictification	II.176
Chapter 7. The Braided Baez Conjecture	II.181
7.1. The 2-Category of Braided Bimonoidal Categories	II.183
7.2. Weakly Initial Braided Bimonoidal Category	II.186
7.3. Bi-Initial Braided Bimonoidal Category	II.192
Chapter 8. Monoidal Bicategorification	II.197
8.1. Matrix Bicategories	II.198
8.2. The Monoidal Identity and the Monoidal Composition	II.205
8.3. The Monoidal Associator and the Monoidal Unitors	II.214
8.4. Matrix Monoidal Bicategories	II.221
Part 2. E_n-Monoidal Categories	II.227
Chapter 9. Ring, Bipermutative, and Braided Ring Categories	II.229
9.1. Ring Categories	II.232
9.2. Endomorphism Ring Categories	II.237
9.3. Elmendorf-Mandell Bipermutative Categories	II.244
9.4. Reduction of Bipermutative Category Axioms	II.247
9.5. Braided Ring Categories	II.252
9.6. Ring Categorical Drinfeld and Symmetric Centers	II.255
9.7. Notes	II.257
Chapter 10. Iterated and E_n -Monoidal Categories	II.259
10.1. Iterated Monoidal Categories	II.264
10.2. Two-Fold Monoidal Categories from Totally Ordered Monoids	II.269
10.3. Iterated Monoidal Functors	II.272
10.4. Monoids in Iterated Monoidal Categories	II.279
10.5. Free Iterated Monoidal Categories	II.284
10.6. Coherence of Iterated Monoidal Categories	II.293
10.7. E_n -Monoidal Categories	II.295
10.8. Braided Ring Categories are E_2 -Monoidal Categories	II.299
10.9. Bipermutative Categories are E_n -Monoidal Categories	II.302
10.10. Free E_n -Monoidal Categories	II.303
10.11. Notes	II.306