Introduction

This book is based on several courses that I taught at the University of Cincinnati. Chapters 1–4 are based on the course "Differential Equations" for sophomores in science and engineering. Only some basic concepts of multivariable calculus are used (functions of two variables and partial derivatives), and they are reviewed in the text. Chapters 7 and 8 are based on the course "Fourier Series and PDE", and they should provide a wide choice of material for the instructors. Chapters 5 and 6 were used in graduate ODE courses, providing most of the needed material. Some of the sections of this book are outside of the scope of usual courses, but I hope they will be of interest to students and instructors alike. The book has a wide range of problems.

I attempted to share my enthusiasm for the subject and write a textbook that students will like to read. While some theoretical material is either quoted or just mentioned without proof, my goal was to show all of the details when doing problems. I tried to use plain language and not to be too wordy. I think that an extra word of explanation has often as much potential to confuse a student as to be helpful. I also tried not to overwhelm students with new information. I forgot who said it first: "One should teach the truth, nothing but the truth, but not the whole truth."

I hope that experts will find this book useful as well. It presents several important topics that are hard to find in the literature: Massera's theorem, Lyapunov's inequality, Picone's form of Sturm's comparison theorem, the "sideways" heat equation, periodic population models, the "hands on" numerical solution of nonlinear boundary value problems, the isoperimetric inequality, etc. The book also contains new exposition of some standard topics. We have completely revamped the presentation of the Frobenius method for the series solution of differential equations, so that the "regular singular points" are now hopefully in the past. In the proof of the existence and uniqueness theorem, we replaced the standard Picard iterations with monotone iterations, which should be easier for students to absorb. There are many other fresh touches throughout the book. The book contains a number of interesting nonstandard problems, including some original ones, published by the author over the years in the Problem Sections of *SIAM Review*, *Electronic Journal of Differential Equations*, and other journals. All of the challenging problems are provided with hints, making them easy to solve for instructors. We use an asterisk (or star) to identify nonstandard problems and sections.

How important are differential equations? Here is what Isaac Newton said: "It is useful to solve differential equations." And what he knew was just the beginning. Today differential equations are used widely in science and engineering. This book presents many applications as well. Some of these applications are very old, like the tautochrone problem considered by Christian Huygens in 1659. Some applications,

like when a drone is targeting a car, are modern. Differential equations are also a beautiful subject, which lets students see calculus "in action".

I attempted to start each topic with simple examples, to keep the presentation focused, and to show all of the details. I think this book is suitable for self-study. However, instructor can help in many ways. He (she) will present the subject with the enthusiasm it deserves, draw more pictures, talk about the history, and tell jokes that supplement the lame ones in the book.

I am very grateful to the MAA Book Board, including Steve Kennedy, Stan Seltzer, and the whole group of anonymous reviewers, for providing me with detailed lists of corrections and suggested changes. Their help was crucial in making considerable improvements to the manuscript.

It is a pleasure to thank Ken Meyer and Dieter Schmidt for constant encouragement while I was writing this book. I also wish to thank Ken for reading the entire book and making a number of useful suggestions, like doing Fourier series early, with applications to periodic vibrations and radio tuning. I wish to thank Roger Chalkley, Tomasz Adamowicz, Dieter Schmidt, and Ning Zhong for a number of useful comments. Many useful comments also came from students in my classes. They liked the book, and that provided me with the biggest encouragement.