# Preface

## To students

We hope you enjoy this book and find it useful. Those who understand interest theory can be informed borrowers, making intelligent choices about mortgages and other loans, and they can also be wise investors. We anticipate that the majority of our readers will be interested in exploring actuarial careers. However, among our enthusiastic students, we have counted many who just felt that this was material they would like to understand; they found it interesting and applicable. Interest rates affect us all! The authors have frequently been told that the course for which the book was designed was a student's "favorite" in his or her college career — and this was not just by students in the University of Texas' actuarial program! With this in mind, we have tried to write a book that will be appealing to any student who wishes to become familiar with how investments grow over time, and who appreciates this being carried out in a mathematically precise manner.

If you are embarking upon a career as an actuary, a mastery of interest theory will be very important for you. This is reflected in the fact that the Society of Actuaries (SOA) and the Casualty Actuarial Society (CAS) each require their Associates to pass an exam including interest theory. Our text is designed to meet the needs of individuals hoping to master this material. Should you work in other financial areas, we believe that an exposure to the rigorous introduction presented in this text will be advantageous.

We were trained as mathematicians, specializing in algebraic number theory, algebraic geometry, and numerical analysis, respectively. We learned interest theory by teaching the subject. Not being insiders in the world of finance, we were like you, the readers for whom this book has been written. Sure, we have a great deal more mathematical sophistication and experience than we would expect of our student readers — but, like you, we found ourselves searching for the larger context in which to place the material. It is one thing to be adept at handling equations, another to understand why it is useful to make these calculations. We hope that our struggles to see the big picture, and the questions posed to us by our students over the roughly forty terms that we have taught interest theory, have helped us to write a book in which each concept is well motivated and described in a manner that leaves the reader feeling satisfied.

While we have striven to write a student-friendly book, helpful to those trying to learn interest theory on their own, as well as to students in organized classes, we want to warn you that this is not easy material. In fact, the level of difficulty may be deceptive. The majority of the problems describe a financial

arrangement in words — producing the dreaded "word problem." To solve the problems, you must extract all the information contained in each sentence. Sometimes this is accomplished by writing equations, but at other times it would be best to use a diagram or list. It is at this stage that many students of interest theory have difficulty, but we are committed to helping our readers develop their problem-solving skills.

Patience, attention to details, and a willingness to work hard in an organized manner are all needed to be a successful student of interest theory. Along with a strong foundation in algebra, these qualities are the most important prerequisite. Recognizing this, we have written this book so that an able student who has not yet studied calculus can follow along. There are a few places where calculus is used, but an alternate path is given for those yet to learn calculus.

## Examples

A careful description of the theory is important; however, it is just as important that there be many examples presented in detail. The text includes more than 260 examples. Each was written, then revised, with the questions: "Have we given an explanation that is clear to a reader who was not previously adept at working such a problem?" and "Have we described why the path we have taken is a natural and good one?" We believe that complicated examples should be preceded by simple ones, and that it is often helpful to present more than one way to obtain the solution.

## Problems

Of course, no matter how helpful our examples may be, it is important that a student tackles many problems. Like all mathematical endeavors, mastering interest theory requires a willingness to practice and to learn from mistakes. *Mathematical Interest Theory* includes more than 475 problems. Just as is the case with our examples, these range from the straightforward to the challenging. Some problems may be completed quickly, while others involve significant analysis followed by considerable calculation. There is an appendix in which numerical (and other short answers) are given. Detailed solutions to the odd-numbered problems are left for the author-prepared student solution manual.

## Special Features

Most of the problems involve computation, and naturally we wish to provide our readers with information on how to effectively utilize technology. We have elected to include considerable discussion and key-by-key examples for our readers who have access to Texas Instruments BAII Plus or BAII Plus Professional calculators. Chapter 0 introduces the reader to these calculators. The

calculator discussion is typeset in a different font from the rest of the text, making the book easy to use, whether or not a reader wants to consider the instructions for these calculators.

It is easy for students of interest theory to feel overwhelmed by the number of formulas. We therefore have placed the equations that are especially important in boxes. Readers may decrease the number of equations they need to memorize if they note which ones can quickly be derived from another formula they have already learned. Statements placed in boxes are also especially important.

We hope that our book will be considered by teachers wishing to offer a class with a significant writing component, but not a course on writing proofs. The end-of-chapter writing problems should be very useful to instructors of such courses.

## Coverage

We present a classic introduction to interest theory in Chapters 1 through 5. However, we introduce yield rates at an earlier stage than is usual, then revisit them as we introduce new financial settings. Students often have difficulty with yield rates, and we believe that introducing them early and using them regularly goes a long way to help students with a topic that many find difficult.

Bonds are corporate and government loans, and have their own language and conventions. The mathematics of these are discussed in Chapter 6. Factors that influence yields for bonds are discussed in Section 10.6. If a company wishes to raise money without borrowing money, it may do so by issuing equity shares or stocks. Stocks are considered in Chapter 7.

The financial horizon has changed markedly over the last half century. It is now important for actuaries and others working in financial services, not to mention those following the news headlines, to understand financial derivatives such as options and futures. In the aftermath of Hurricane Katrina, gas futures made the headlines. Chapter 8 introduces in detail modern financial concepts such as arbitrage, derivatives, options, futures, and swaps, and it does so without requiring the reader to know probability theory.

Risk management is important in a world where interest rates may change in a manner that is difficult to foresee. In Chapter 9 we introduce the reader to asset-liability matching and to immunization.

While much of the focus in an interest theory course is on performing computations based on given information, Chapter 10 addresses the factors that determine the level of interest rates.

There is more material in this book than is apt to be covered in a one-semester undergraduate course. For many courses, the syllabus will likely be determined to coincide with the syllabus for the SOA's Exam FM. The first and second editions of *Mathematical Interest Theory* have been listed by the

examination committee as appropriate reading, and this third edition has been endorsed as well. Modifications in the precise SOA syllabus occur frequently, so interested parties should check the SOA website for current coverage. The sections of the first and second edition that have been suggested are Chapter 1 [(1.3) – (1.12), (1.14)], Chapter 2 [(2.2) – (2.7)], Chapter 3 [(3.2) – (3.9), (3.11), (3.13)], Chapter 4 [(4.2) – (4.6)], Chapter 5 [(5.2), (5.4)], Chapter 6 [(6.2) – (6.9)], Section (7.1), Section (8.3), and Chapter 9 [(9.1) – (9.4)]. The corresponding sections in the third edition are Chapter 1 [(1.3) – (1.12), (1.15)], Chapter 2 [(2.2) – (2.7)], Chapter 3 [(3.2) – (3.9), (3.11), (3.13)], Chapter 4 [(4.2) – (4.6)], Chapter 5 [(5.2), (5.4)], Chapter 6 [(6.2) – (6.9)], Section (7.1), Section (8.3), and Chapter 9 [(9.1) – (9.3), (9.6)].

If a teacher's focus is not governed by a desire to cover the interest theory needed on actuarial exams, the choice of material will depend on individual preferences and the backgrounds of the students. However, we suggest all readers carefully study Chapters 1–3, except possibly for sections requiring calculus.

## Third edition

In the summer of 2017, changes made by the SOA to Exam FM's syllabus took effect. The third edition includes all of the "new" topics listed in its learning objectives. Section (1.14) introduces the quoted rates of the United States and Canadian government's treasury bills. After a survey of effective rates, nominal rates, and force of interest, a brief discussion on why one might choose one method of quoting an interest rate over another is given in Section (1.16). While most derivatives were removed from Exam FM's syllabus, the treatment of interest rate swaps was expanded. An introduction to loans with floating rate of interest is given in Section (8.4), followed by interest rate swaps in Sections (8.5)—(8.7). First and second-order Macaulay approximations and modified approximations are shown in Sections (9.4) and (9.5). A survey of factors that determine the level of interest rates is given in Chapter 10. All topics from the second edition have been retained for the third edition, but some of the section numbers have been changed to accommodate the new sections. Currency swaps and total return equity swaps have been separated from interest rate swaps and moved to Section (8.8).

Another change was made to better align the vocabulary in *Mathematical Interest Theory* with the terminology used in Exam FM. The simple interest approximation to the yield rate, which was called the "approximate dollar-weighted yield rate" in the second edition, has been renamed the "dollar-weighted yield rate" in Section (2.6) of the third edition.

Other changes have been made, based on the many conversations we have shared with our students.

A new example, Example (1.7.3), has been inserted between what used to be Examples (1.7.2) and (1.7.3) in the second edition. The solution for what was Example (1.7.3) (Example (1.7.4) in the third edition) has been rewritten. Hopefully this better illuminates how the accumulation function and discount function are used when money is invested at a time other than zero.

The solutions for Example (9.2.24), duration of a coupon bond purchased at par, and Example (9.2.26), duration of an amortized loan with no early repayment option, have been modified to use only annuity formulas from Chapter 3. This was done with the hope that readers who have not seen Chapter 4 in its entirety will study these examples and find them useful.

Exercises are provided for the newly added sections and a few more have been added to existing sections such as Sections (2.4) and (8.3). Some exercises have been modified from the second edition.

Finally, the third edition is a revision by Shinko Harper of *Mathematical Interest Theory* by Leslie Vaaler and James Daniel.

## Financial transactions

This book includes accounts of numerous financial arrangements. For the most part, these are fictional examples, designed to illustrate and reinforce general principles. We made up the names of participants and financial institutions and any overlap with the names of actual individuals or businesses is accidental. You will notice that the names have diverse ethnic roots. We hope that no gender or nationality appears too often as a borrower or unwise investor.

We have already mentioned that we are mathematicians rather than financial advisors. However, at this point we are duty-bound to give you one piece of financial advice. *Always* make sure you understand all details of any financial arrangement *before* you agree to it. Ask questions. Do not rely on everything working as it does in this book.

## Acknowledgments

We offer heartfelt thanks to our students. Their questions, comments, and even their mistakes were helpful to us as we wrote and revised this book. Without our students, there would have been no book.

Carl Gillette, Karen Kimberly, and Gagan Nanda worked through the examples and problems as accuracy checkers for the first edition. For her role in the third edition, we add Holly Merrell to this list. Appreciation is due to each for their invaluable work.

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Thanks to the University Coop for honoring the first edition of this book with a Robert W. Hamilton Book Award. It is heartening to know that a committee of non-mathematicians selected our book from a large group of books with University of Texas authors.

Once again, we thank our families and friends. Good investments stand the test of time!

## Contacting the authors

If you note errors of any sort, we would appreciate you informing us of them. Other comments and suggestions for future editions are also welcome. Contact us at `shinko@math.utexas.edu`.

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