1

Calculating on the Back of an Envelope

In this first chapter we learn how to think about questions that need only good enough answers. We find those answers with quick estimates that start with reasonable assumptions and information you have at your fingertips. To make the arithmetic easy we round numbers drastically and count zeroes when we have to multiply.

Chapter goals:

Goal 1.1. Verify quantities found in the media, by checking calculations and with independent web searches.

Goal 1.2. Estimate using common sense and common knowledge.

Goal 1.3. Learn about the Google calculator (or another internet calculator).

Goal 1.4. Round quantities to report only an appropriate number of significant digits.

Goal 1.5. Learn when not to use a calculator — become comfortable with quick approximate mental arithmetic.

Goal 1.6. Work with large numbers.

Goal 1.7. Work with (large) metric prefixes

Goal 1.8. Practice with straightforward unit conversions.

1.1 Hailing a ride

On May 1, 2018, a headline in *The Boston Globe* read "There were nearly 100,000 Uber and Lyft rides per day in Boston last year." The article began:

> It's not your imagination: There are an awful lot of Uber and Lyft cars in Boston traffic.
>
> The ride-hailing companies provided nearly 35 million trips in Boston in 2017, or an average of about 96,000 every day, according to data released by Massachusetts officials Tuesday.
>
> Boston accounted for more than half of the 65 million rides Uber and Lyft provided across all of Massachusetts last year, according to data the Department of Public Utilities collected from the ride-hailing companies under a new state law regulating the industry. Every minute in the city there were, on average, 67 Uber and Lyft rides underway. [**R3**]

There are several numbers here: 100,000, 35 million, 96,000, 65 million, and 67. Do these numbers make sense?

In particular, should you believe Uber and Lyft accounted for "35 million trips in Boston in 2017"?

Note the question. "Should you believe?" not "Do you believe?". To decide whether or not to believe we will think about what that number says in a context where you can see whether (or not) it makes common sense. Imagining a year's worth of trips is hard. Trips per day or per hour or per minute might be easier.

Start by checking the arithmetic in the second paragraph. If we work with 350 days in a year it's easy to divide the 35 million rides among the days. Cancelling the 35 and counting zeroes gives 100,000 rides per day. That's exactly what's in the headline and is for all common sense purposes the same as the 96,000 daily rides in the quotation.

We could have used a calculator to discover that

$$\frac{35,000,000}{365} = 95890.4109589.$$

But why waste the time? All we know to start with is that the numerator is "nearly 35,000,000". That number has just two significant digits, the 3 and the 5. The six zeroes just tell us where the decimal point goes. So the answer should be rounded to two significant digits: about 96,000 daily rides. That is just what the author of the article did. All the other digits are correct arithmetically but make no sense in the discussion.

Should you believe 100,000 rides per day? Let's suppose that the people who use Uber or Lyft use it twice a day. That's two rides per person, so that the 100,000 rides per day are taken by 50,000 people. About 1,000,000 people live in the Boston metropolitan area. Perhaps one in twenty takes a ride-hail round trip. This seems reasonable.

You can argue about some of the assumptions. 50,000 is probably not a very good estimate for the number of riders (each twice). But it does tell us that 100,000 rides per day is in the right ballpark. It has the right number of zeroes. 10,000 rides per day would clearly be too small while 1,000,000 rides per day would be too large.

Another way to look at 100,000 rides per day is to think about the number of rides per hour, and then per minute. There are about 25 hours in a day, so about 4,000 rides per hour. There are 60 minutes in an hour, and 40/6 is about 7, so there are about 70 rides per minute. That's believable if you imagine people all over Boston looking for rides.

We've checked that the 35 million rides per year is a reasonable number. Should we believe 67 rides underway at any particular time? It feels too small. We just estimated about 70 rides starting per minute. If each ride lasted just one minute there would be 70 rides underway at any time. So 67 can't be right.

The last paragraph of the article provides some data to confirm our suspicion.

> The average ride-hailing trip in Massachusetts lasts about 15 minutes and travels about 17 miles an hour, according to the state data. That's heavily influenced by the clustering of trips in Boston, where the average ride-hailing trip travels at 16 miles an hour.

A 15 minute ride at 16 miles per hour is a 4 mile ride. That's probably about right for a trip in the city, or to or from a suburb.

Since the average ride lasts 15 minutes and 70 rides start each minute there must be about $15 \times 70 \approx 1,000$ rides underway (on average) at any time. That makes much more sense than 67. Should we believe it? Let's look for more evidence.

Googling "how many Uber drivers in Boston" finds this quote from 2015 at `www.americaninno.com/boston/google-ride-hailing-service-google-uber-may-be-competitors-2-2/`:

> There are nearly 10,000 Uber drivers in Boston, according to data the ridesharing company released Thursday morning. [**R4**]

You can't believe everything you read on the internet, but this site says the 10,000 driver figure comes from Uber. Uber might want to exaggerate, but it's probably an OK estimate. That was several years before the 2017 report. Updating and adding Lyft drivers suggests there might have been 20,000 drivers in 2017. If 5 percent of them had passengers at any moment, that would account for a believable 1,000 rides on the road.

Where might the 67 have come from? A clue is how close it is to our estimate that about 70 rides start each minute. In fact it's just what you get when you carefully convert 96,000 rides per day into

$$\frac{96,000 \text{ rides}}{\text{day}} \times \frac{1 \text{ day}}{24 \text{ hours}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = \frac{66.666\ldots \text{ rides}}{\text{minute}}.$$

So the author meant to say

> Every minute there were, on average, 67 Uber and Lyft rides *starting*.

rather than

> Every minute there were, on average, 67 Uber and Lyft rides *underway*.

If we had discovered this error when the article first appeared we could have written the author asking for a correction in the online story.

What have we learned? First, that almost all the numbers in this article are consistent with one another: they fit together. Only the 67 rides on the road is wrong. Second, though 35,000,000 Boston rides per year might at first seem unbelievably large, it is in fact reasonable. Finally, thinking with the numbers rather than skipping over them led us to discover and correct a mistake and helped us focus on the ideas that the article is meant to convey.

1.2 How many seconds?

Have you been alive for a thousand seconds? A million? A billion? A trillion?

Before we estimate, what's your guess? Write it down, then read on.

To turn a guess into an estimate you have to do some arithmetic. There are two ways to go about the job. You can start with seconds and work up through hours, days and years, or start with thousands, millions and billions of seconds and work backwards to hours, days and years. We'll do it both ways.

How many seconds in an hour? Easy: $60 \times 60 = 3{,}600$. So we've all been alive much more than thousands of seconds.

Before we continue, we're going to change the rules for arithmetic so that we can do all the multiplication in our heads, without calculators or pencil and paper. We will round numbers so that they start with just one nonzero digit, so 60×60 becomes 4,000. Of course we can't say $60 \times 60 = 4,000$; the right symbol is $\approx$, which means "is approximately." We call this "curly arithmetic".

Then an hour is

$$60 \times 60 \approx 4{,}000$$

seconds.

There are 24 hours in a day. $4 \times 24 \approx 100$, so there are

$$4,000 \times 24 \approx 100,000$$

seconds in a day.

Or we could approximate a day as 20 hours, since we overestimated the number of seconds in a hour. That would mean (approximately) 80,000 seconds per day. We'd end up with the same (approximate) answer.

Since there are about a hundred thousand seconds in a day, there are about a million seconds in just 10 days. That's not even close to a lifetime, so we'll skip working on days, weeks or months and move on to years.

How many seconds in a year? Since there are (approximately) 100,000 in a day and (approximately) 400 days in a year there are about 40,000,000 (forty million) seconds in a year.

If we multiply that by 25 the 4 becomes 100, so a 25 year old has lived for about 1,000,000,000 (one billion) seconds.

Does this match the estimate you wrote down for your lifetime in seconds?

A second way to estimate seconds alive is to work backwards. We'll write the time units using fractions — that's looking ahead to the next chapter — and round the numbers whenever that makes the arithmetic easy. Let's start with 1,000 seconds.

$$\begin{aligned}
1{,}000\ \cancel{\text{seconds}} \times \frac{1\ \text{minute}}{60\ \cancel{\text{seconds}}} &= \frac{1{,}000}{60}\ \text{minutes} \\
&= \frac{100}{6}\ \text{minutes (cancel a 0)} \\
&= \frac{50}{3}\ \text{minutes (cancel a 2)} \\
&\approx \frac{60}{3}\ \text{minutes (change 50 to 60 — make division easy)} \\
&= 20\ \text{minutes.}
\end{aligned}$$

We're all older than that.

How about a million seconds? A million has six zeroes — three more than 1,000, so a million seconds is about 20,000 minutes. Still too many zeroes to make sense of, so convert to something we can understand — try hours:

$$20,000\,\cancel{\text{minutes}} \times \frac{1\text{ hour}}{60\,\cancel{\text{minutes}}} = \frac{20,000}{60}\text{ hours } = \frac{1,000}{3}\text{ hours } \approx 300\text{ hours.}$$

There are 24 hours in a day. To do the arithmetic approximately, use 25. Then $300/25 = 12$ so 300 hours is about 12 days. We've all been alive that long.

How about a billion seconds? A billion is a thousand million, so we need three more zeroes. We can make sense of that in years:

$$12,000\,\cancel{\text{days}} \times \frac{1\text{ year}}{365\,\cancel{\text{days}}} = \frac{12,000}{365}\text{ years } \approx \frac{12,000}{400}\text{ years } \approx 30\text{ years.}$$

Since a billion seconds is about 30 years, it's in the right ballpark for the age of most students. It's in the same ballpark as the 25 year estimate we found doing the arithmetic the other way around.

A trillion is a thousand billion — three more zeroes. So a trillion seconds is about 30,000 years. Longer than recorded history.

1.3 Heartbeats

In *The Canadian Encyclopedia* a blogger noted that

> The human heart expands and contracts roughly 100,000 times a day, pumping about 8,000 liters of blood. Over a lifetime of 70 years, the heart beats more than 2.5 billion times, with no pit stops for lube jobs or repairs. [**R5**]

Should we believe "100,000 times a day" and "2.5 billion times in a lifetime"?

If you think about the arithmetic in the previous section in a new way, you may realize we just answered this question. Since your pulse rate is about 1 heartbeat per second, counting seconds and counting heartbeats are different versions of the same problem. We discovered that there are about 100,000 seconds in a day, so the heartbeat count is about right. We discovered that 30 years was about a billion seconds, and 70 is about two and a half times 30, so 70 years is about 2.5 billion seconds. Both the numbers in the article make sense.

Even if we didn't know whether 100,000 heartbeats in a day was the right number, we could check to see if that number was consistent with 2.5 billion in a lifetime.

To do that, we want to calculate

$$100,000\frac{\text{beats}}{\text{day}} \times 365\frac{\text{days}}{\text{year}} \times 70\frac{\text{years}}{\text{lifetime}}.$$

Since we only need an approximate answer, we can simplify the numbers and do the arithmetic in our heads. If we round the 365 up to 400 then the only real multiplication is $4 \times 7 = 28$. The rest is counting zeroes. There are eight of them, so the answer is approximately $2,800,000,000 = 2.8$ billion. That means the 2.5 billion in the article is about right. Our answer is larger because we rounded up.

The problems we've tackled so far don't have exact numerical answers of the sort you are used to. The estimation and rounding that goes into solving them means that when you're done you can rely on just a few *significant digits* (the digits at the beginning of a number) and the number of zeroes. Often, and in these examples in particular, that's all you need. Problems like these are called "Fermi problems" after Enrico Fermi (1901–1954), an Italian physicist famous for (among other things) his ability to estimate the answers to physical questions using very little information.

1.4 Calculators

The thrust of our work so far has been on mental arithmetic. You can always check yours with the calculator on your phone. But that requires clicking number and operator icons. If you have internet access, Google's is easier to use — simply type

```
100,000 * 365 * 70
```

into the search box to find the number of heartbeats in a lifetime, assuming 100,000 in a day. Google displays a calculator showing

```
2555000000
```
.

That 2.555 billion answer is even closer than our first estimate to the 2.5 billion approximation in the article. The Bing search engine offers the same feature.

You can click on the number and operation keys in the Google calculator on display to do more arithmetic. Please don't. Use the keyboard rather than the mouse. It's faster, and you can fix typing mistakes easily.

The Google calculator can do more than just arithmetic — it can keep track of units. Although it doesn't count heartbeats, it does know about miles and about speeds like miles per day and miles per year. We can make it do our work for us by asking about miles instead of heartbeats. Search for

```
100,000 miles per day in miles per 70 years
```

and Google rewards you with

$$\boxed{100\,000 \text{ (miles per day)} = 2.55669539 \times 10^9 \text{ miles per (70 years)}}\ .$$

The "$\times 10^9$" means "add nine zeroes" or, in this case, "move the decimal point nine places to the right", so

$$\begin{aligned} 100,000 \text{ (miles per day)} &= 2.55669539 \times 10^9 \text{ miles per (70 years)} \\ &\approx 2.6 \text{ billion miles per (70 years).} \end{aligned}$$

That is again "more than 2.5 billion."

The exact answer from Google is even a little more than the 2,555,000,000 we found when we did just the arithmetic since Google knows a year is a little longer than 365 days — that's why we have leap years.

So 100,000 heartbeats per day does add up to about 2.5 billion in 70 years. We've checked that the numbers are consistent — they fit together.

But are they correct? Does your heart beat 100,000 times per day? To think sensibly about a number with lots of zeroes we can convert it to a number of something equivalent with fewer zeroes — in this case, heartbeats per minute. That calls for division rather than multiplication:

$$100,000 \frac{\text{beats}}{\text{day}} \times \frac{1 \text{ day}}{24 \text{ hours}} \times \frac{1 \text{ hour}}{60 \text{ minutes}}.$$

To do the arithmetic in your head, round the 24 to 25. Then $25 \times 6 = 150$ — there are about 1,500 minutes in a day. Then $100,000/1,500 = 1,000/15$. Since 100/15 is about 7, we can say that $1,000/15$ is about 70. 70 beats per minute is a reasonable estimate for your pulse rate, so 100,000 heartbeats per day is about right.

Google tells us

$$\boxed{\text{100 000 (miles per day)} = \text{69.4444444 miles per minute}}\ .$$

The nine and all the fours in that 69.4444444 are much too precise. The only sensible thing to do with that number is to round it to 70 — which is what we discovered without using a calculator.

Sometimes even the significant digits can be wrong and the answer right, as long as the number of zeroes is correct. Informally, that's what we mean when we say the answer is "*in the right ballpark*." The fancy way to say the same thing is "the *order of magnitude* is correct." For example, it's right to say there are hundreds of days in a year — not thousands, not tens. There are billions (nine zeroes) of heartbeats in a lifetime, not hundreds of millions (eight zeroes), nor tens of billions (ten zeroes).

1.5 Millions of trees?

On May 4, 2010, Olivia Judson wrote in *The New York Times* [**R6**] about Baba Brinkman, who describes himself on his webpage as

> . . . a Canadian rap artist, award-winning playwright, and former tree-planter who has personally planted more than one million trees. [**R7**]

How long would it take to personally plant a million trees? Is Brinkman's claim reasonable?

To answer that question you need two estimates — the time it takes to plant one tree and the time Brinkman may have spent planting.

To plant a tree you have to dig a hole, put in a seedling and fill in around the root ball. It's hard to imagine you can do that in less than half an hour.

If Brinkman worked eight hours a day he would plant 16 trees per day. Round that up to 20 trees per day to make the arithmetic easier and give him the benefit of the doubt. At that rate it would take him 1,000,000/20 = 50,000 days to plant a million trees. If he planted trees 100 days each year, it would take him 500 years; if he planted trees for 200 days out of the year, it would take him 250 years. So his claim looks unreasonable.

What if we change our estimates? Suppose he took just ten minutes to plant each tree and worked fifteen hour days. Then he could plant nearly 100 trees per day. At that rate it would take him 10,000 days to plant a million trees. If he worked 100 days each summer he'd still need about 100 years. So on balance we believe he's planted lots of trees, but not "personally . . . more than one million."

It's the "personally" that makes this very unlikely. We can believe the million trees if he organized tree-planting parties, perhaps with people manning power diggers of some kind, or if planting acorns counted as planting trees.

This section, first written in 2010, ended with that unfunny joke until 2013, when Charles Wibiralske, teaching from this text, wondered if we might be overestimating the time it takes to plant a tree. To satisfy his curiosity, he found Brinkman's email address and asked. The answer was a surprising (to him and to us) ten seconds! So our estimate of 10 minutes was 60 times too big. That means our 100 year estimate should really have been only about two years! That's certainly possible. If it took him a minute per tree rather than 10 seconds he could still have planted a million trees in several summers.

Brinkman tells the story of Wibiralske's question and this new ending in his blog at `www.bababrinkman.com/insult-to-injury/` . When you visit you can listen to "The Tree Planter's Waltz" (`www.youtube.com/watch?v=jk-jifbpcww`).

The moral of the story: healthy skepticism about what you read is a good thing, as long as you're explicit and open minded about the assumptions you make when you try to check. That's a key part of using common sense.

Brinkman's blog ends this way:

> Hurray! In the end it's a classic example of … the drunkard's walk towards knowledge. When our views are self-correcting and open to revision based on new evidence, they will continue to hone in on increasingly accurate representations of the real world. That's good honest skepticism, and when it wins over bad, knee-jerk, "it's hard to imagine" skepticism, that's a beautiful thing.

1.6 Carbon footprints

Discussions about global warming and climate change sometimes talk about the *carbon footprint* of an item or an activity. That's the total amount of carbon dioxide (CO_2) the item or activity releases into the atmosphere. An article in *The Boston Globe* on October 14, 2010, listed estimates of carbon footprints for some common activities. Among those was the 210 gram carbon footprint of a glass of orange juice. That includes the carbon dioxide cost of fertilizing the orange trees in Florida and harvesting the oranges and the carbon dioxide generated burning oil or coal to provide energy to squeeze the oranges, concentrate and freeze the juice and then ship it to its destination. It's just an estimate, like the ones we're learning to make, but much too complex to ask you to reproduce. We drew Figure 1.1 using the rest of the data from the article.

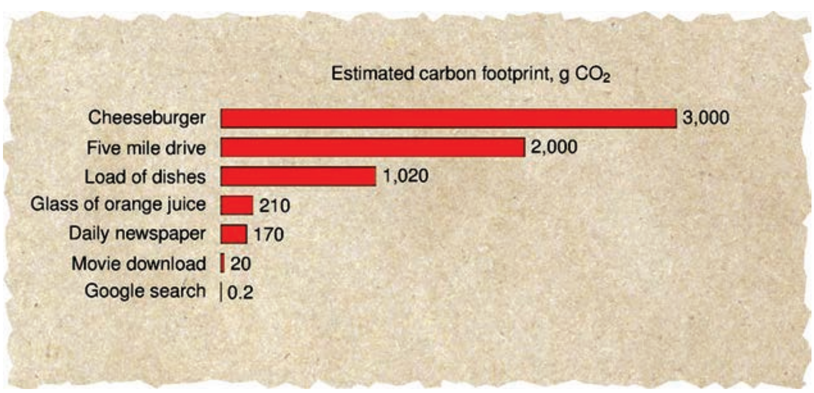

Figure 1.1. Carbon footprints [**R8**]

Let's look at the orange juice. How many glasses are consumed in the United States each day? The population of the United States is about 330 million. To make the arithmetic easy, round that down to 300 million. If we estimate that about 5% of the population has orange juice for breakfast that means 15 million glasses. So the ballpark answer is on the order of 10 to 20 million glasses of orange juice. Since each glass contributes 200 grams, 10 to 20 million glasses contribute 2 to 4 billion grams each day.

It's hard to imagine 2 billion grams. You may have heard of kilograms — a kilogram is about two pounds. In the metric system *kilo* means "multiply by 1,000" so a kilogram is 1,000 grams. Then 2 billion grams of carbon is just 2 million kilograms. That's about 4 million pounds, or 2 thousand tons.

Since there are seven activities listed in the graphic, there are six other Fermi problems like this one for you to work on:

- Google search
- Movie download
- Daily newspaper printed
- Dishwasher run
- Five miles driven
- Cheeseburger consumed

For each you can estimate the total number of daily occurrences and then the total daily carbon contribution. We won't provide answers here because we don't want to spoil a wonderful class exercise.

1.7 Kilo, mega, giga

Counting zeroes is often best done three at a time. That's why we separate groups of three digits by commas. Each step from thousands to millions to billions adds three zeroes. The metric system has prefixes for that job.

We've seen that *kilo* means "multiply by 1,000". Similarly, *mega* means "multiply by 1,000,000". A Megabucks lottery prize is millions of dollars. A megathing is 1,000,000 things, whatever kind of thing you are interested in.

New Hampshire's Seabrook Nuclear Power Plant is rated at 1,270 megawatts. So although you may not know what a watt is, you know this power plant can generate 1,270,000,000 of them. The symbols for "mega" and for "watt" are "M" and "W" so you can write 1,270 megawatts as 1,270 MW.

How many 100 watt bulbs can Seabrook light up? Just take the hundred's two zeroes from the mega's six, leaving four following the 1,270. Putting the commas in the right places, that means 12,700,000 bulbs. Or, if you want to be cute, about 13 megabulbs.

Next after mega is *giga*: nine zeroes. The symbol is "G". When you say "giga" out loud the G is hard, even though it's soft in the word "gigantic".

You could describe Seabrook as a 1.27 gigawatt power plant.

Table 1.2 describes the metric prefixes bigger than giga. There's no need to memorize it. You will rarely need the really big ones. You can look them up when you do.

The metric system also has prefixes for shrinking things as well as these for growing them. Since division is harder than multiplication, we'll postpone discussing those prefixes until we need them, in Section 2.7.

Name	Symbol	Meaning	English name	Zeroes
Kilo	K	$\times 10^3$	thousand	3
Mega	M	$\times 10^6$	million	6
Giga	G	$\times 10^9$	billion	9
Tera	T	$\times 10^{12}$	trillion	12
Peta	P	$\times 10^{15}$	quadrillion	15
Exa	E	$\times 10^{18}$	quintillion	18
Zetta	Z	$\times 10^{21}$	sextillion	21
Yotta	Y	$\times 10^{24}$	septillion	24

Table 1.2. Metric prefixes

1.8 Exercises

Notes about the exercises:

- The Preface to the First Edition has information about the exercises and the solutions, including explanations of the bracketed notations like [S] that accompany each exercise.

- One of the ways to improve your quantitative reasoning skills is to write about what you figure out. The exercises give you many opportunities to practice that. The answer to a question should be more than just a circle around a number or a simple "yes" or "no". Write complete paragraphs that show your reasoning. Your answer should be complete enough so that you can use a corrected homework paper to study for an exam without having to go back to the text to remember the questions.

- It often helps to write about doubt and confusion rather than guessing at what you hope will turn out to be a right answer.

- To solve Fermi problems you make assumptions and estimates. It's those skills we are helping you develop. The web can help, but you should not spend a lot of time searching for answers to the particular questions we ask. That's particularly true since many of the problems don't have a single right answer.

- Be sure to make your estimates and assumptions explicit. Then you can change them easily if necessary, as we did in Section 1.5.

- In some of the chapters the interesting exercises are followed by a few routine exercises where you can practice just the arithmetic, independent of any real applications.

Your instructor has a Solutions Manual with answers to the exercises, written the way we hope you will write them. Ask him or her to provide some for you.

Exercise 1.8.1. [S][Section 1.3][Goal 1.6] Warren Buffet is very rich.

In *The Boston Globe* on September 30, 2015, you could read that Warren Buffet is worth $62 billion, and that

> ...if [he] gave up on aggressive investing and put his money into a simple savings account, with returns at a bare 1 percent, he'd earn more in interest each hour than the average American earns in a year. [**R9**]

Use the data in this story to estimate the annual earnings of the average American. Do you think the estimate is reasonable?

Exercise 1.8.2. Dropping out.

Moved to Extra Exercises at `www.ams.org/bookpages/text-63`.

Exercise 1.8.3. [S][Section 1.3][Goal 1.1] [Goal 1.2][Goal 1.5][Goal 1.8] Bumper sticker politics.

In the fall of 2015 at `donnellycolt.com` you could buy a button that said

> Every Minute 30 Children Die of Hunger and Inadequate Health Care
> While the World Spends $1,700,000 on War

or a small vinyl sticker with the claim

> Every Minute the World Spends $700,000 on War While 30 children
> Die of Hunger & Inadequate Health Care. [**R10**]

What are these items trying to say? Do the numbers make sense?

Your answer should be a few paragraphs combining information you find on the web (cite your sources — how do you know they are reliable?) and a little arithmetic.

Exercise 1.8.4. [S][Section 1.3][Goal 1.1] [Goal 1.2][Goal 1.4][Goal 1.8] Two million matzoh balls.

In 2012 a modest restaurant in Newton Centre, MA, advertised

Johnny's Luncheonette
Over 2 Million
Matzoh Balls
served!

A year or so later students thinking about whether it was reasonable found that Johnny's website at `www.johnnysluncheonette.com/` claimed "Over 1 Million Matzoh Balls Served!"

What would you believe?

[See the back of the book for a hint.]

Exercise 1.8.5. [S][Section 1.3][Goal 1.2][Goal 1.5] *Writing Your Dissertation in Fifteen Minutes a Day*.

Joan Bolker's book with that title sold about 120,000 copies in the first fifteen years since its publication in 1998.

Estimate the fraction of doctoral students who bought this book.

Exercise 1.8.6. [S][W][Section 1.3][Goal 1.1] [Goal 1.2][Goal 1.4][Goal 1.5] Smartphone apps may help retail scanning catch on.

Some grocery stores are experimenting with a new technology that allows customers to scan items as they shop. Once the customer is done, he or she completes the transaction online and never has to stand in the checkout line. On March 11, 2012, *The Boston Globe* reported that

> Modiv Media's scan-it-yourself technology [is installed] in about 350 Stop & Shop and Giant stores in the United States. Many consumers have embraced the system; Stop & Shop spokeswoman Suzi Robinson said the service handles about one million transactions per month. [**R11**]

(a) Estimate the number of customers per day per store who use this self-scanning technology.

(b) Estimate the number of customers per day per store.

(c) Estimate the percentage of customers who use the technology.

Exercise 1.8.7. Health care costs for the uninsured.

Moved to Extra Exercises at `www.ams.org/bookpages/text-63`.

Exercise 1.8.8. [S][Section 1.2][Goal 1.1][Goal 1.3] Is 25 the same as 30?

In Section 1.2 we showed that a 25 year old has lived for about a billion seconds. Then we estimated that a billion seconds is about 30 years.

(a) Explain why we got two different answers — 25 and 30 years.

(b) Are the answers really different?

(c) Compare them to what the Google calculator says about a billion seconds.

Exercise 1.8.9. [S][Section 1.2][Goal 1.1] [Goal 1.2][Goal 1.4][Goal 1.5] Spoons around the world.

In January 2011 *Whole Foods Magazine* reported that

> According to the Clean Air Council, enough paper and plastic utensils are thrown away every year to circle the equator 300 times. [**R12**]

(a) Estimate the number of utensils it would take to circle the Earth 300 times.

(b) Is the assertion in the quote reasonable?

(c) This 2011 assertion is viral on the internet. Someone writes it once, then it's copied from website to website. What are the earliest and latest versions you can find?

Exercise 1.8.10. [S][Section 1.2][Goal 1.1] [Goal 1.2][Goal 1.3] 1,000,000,000,000,000,000,000,000,000.

On July 25, 2010, Christopher Shea wrote in *The Boston Globe* about "hella", a new metric prefix popular among geeks in northern California.

> Austin Sendek, a physics major at the University of California Davis, wants to take "hella" from the streets and into the lab. With the help of a Facebook-driven public relations campaign, he's petitioning the Consultative Committee on Units, a division of the very serious Bureau International des Poids et Mesures, to anoint "hella" as the official term for a previously unnamed, rather large number: 10 to the 27th power. (The diameter of the universe, by Sendek's reckoning, is 1.4 hellameters.) [**R13**]

(a) Does the Google calculator know about hella?

(b) Does the Bing search engine know about hella?

Exercise 1.8.11. [S][Section 1.2][Goal 1.2] [Goal 1.4][Goal 1.5] Counting fish.

In *To the Top of the Continent* Frederick Cook wrote about this incident in his Alaska travels.

> The run of the hulligans was very exciting ... Mr. Porter's thoughts ran to mathematics, he figured that the train of hulligans was twelve inches wide and six inches deep and that it probably extended a hundred miles. Estimating the number of fish in a cubic foot at ninety-one and one half, he went on to so many millions that he gave it up, suggesting that we try and catch some. [**R14**]

(a) How many millions of hulligans did Mr. Porter try to count?

(b) What's wrong with the precision in this paragraph?

(c) What's a hulligan? Cook's book was published in 1908. Are there any hulligans around today?

Exercise 1.8.12. [S][Section 1.2][Goal 1.1][Goal 1.8] Millions jam street-level crime map website.

In early 2011, the British government introduced a crime-mapping website that allows people to see crimes reported by entering a street name. The launch of the site, however, was problematic. The BBC reported that the website was jammed by up to five million hits per hour, about 75 thousand a minute. [**R15**]

(a) Are the figures "five million an hour" and "75,000 a minute" consistent?

(b) Estimate the fraction of the population of London trying to look at that website. Does your answer make sense?

[See the back of the book for a hint.]

Exercise 1.8.13. The popularity of social networks.

Moved to Extra Exercises at `www.ams.org/bookpages/text-63`.

Exercise 1.8.14. [S][Section 1.2][Goal 1.1][Goal 1.2][Goal 1.5][Goal 1.8] How rich is rich?

On September 19th, 2011, Aaron S. from Florida posted a comment at *The New York Times* in which he claims that if you left one of the 400 wealthy people with just a billion dollars he could not spend his fortune in 30 years at a rate of $100,000 a day. [**R16**]

Is Aaron's arithmetic right? Can you do this calculation without a calculator? Without pencil and paper?

Exercise 1.8.15. Greek debt.

Moved to Extra Exercises at `www.ams.org/bookpages/text-63`.

Exercise 1.8.16. [S][Section 1.3][Goal 1.1] [Goal 1.4][Goal 1.6] No Lunch Left Behind.

From *The New York Times*, Feb 20, 2009, in a column by Alice Waters and Katrina Heron with that headline:

> How much would it cost to feed 30 million American schoolchildren a wholesome meal? It could be done for about $5 per child, or roughly $27 billion a year, plus a one-time investment in real kitchens. [**R17**]

There are three numbers in the paragraph. Are they reasonable? Are they consistent with each other and with other numbers you know?

Exercise 1.8.17. [S][Section 1.3][Goal 1.2] [Goal 1.4][Goal 1.5] Brush your teeth twice a day — but turn off the water.

The Environmental Protection Agency says on its website that

> You can save up to 8 gallons of water by turning off the faucet when you brush your teeth in the morning and before bedtime. [**R18**]

(a) Estimate how much water a family of four would use each week, assuming they left the water running while brushing.

(b) Estimate how much water would be saved in one day if the entire United States turned off the faucet while brushing.

(c) Put your answer to the previous question in context (compare it to the volume of water in a lake or a swimming pool, for example).

(d) Realistically, you need some water to brush your teeth because you need to get the toothbrush wet and you need to rinse the brush and your teeth. Estimate how much water that involves, per brushing, then redo the estimate in part (b).

Exercise 1.8.18. [W][S][Section 1.3][Goal 1.1][Goal 1.2][Goal 1.5] Lady Liberty.

On May 9, 2009, *The Boston Globe* reported that the Statue of Liberty's crown will reopen and that

> 50,000 people, 10 at a time, will get to visit the 265-foot-high crown. [**R19**]

Estimate how long each visitor will have in the crown to enjoy the view.

Exercise 1.8.19. [S][Section 1.3][Goal 1.2] [Goal 1.5] Look ma! No zipper!

The bag of Lundberg Zipper Free California White Basmati Rice advertises

> We've removed the re-closable zipper from our two pound bags, which will save about 15% of the material used to make the bag, which will save 35,000 lbs. of plastic from landfills every year.

(a) How much plastic is still ending up in landfills?

(b) What fact would you need to figure out how many bags of rice Lundberg sells each year? Estimate that number, and then estimate the answer.

Exercise 1.8.20. [S][Section 1.3][Goal 1.4][Goal 1.5][Goal 1.8] Leisure in Peru.

On page 32 in *The New Yorker* on December 7, 2009, Lauren Collins wrote that late arrivals in Peru are said to amount to three billion hours each year. [**R20**]

We suspect that the source of Collins's assertion is an article in the July 1, 2007, edition of *Psychology Today* that commented on the campaign for punctuality and included a feature called "Tardiness by the Numbers" that provided the data:

- 107 hours: annual tardiness per Peruvian
- $5 billion: cost to the country
- 84%: Peruvians who think their compatriots are punctual only "sometimes" or "never"
- 15%: think tardiness is a local custom that doesn't need fixing [**R21**]

(a) According to Collins, how late are Peruvians, in hours per person per day?

(b) Is your answer to the previous question consistent with the numbers in the *Psychology Today* article?

(c) Is the $5 billion "cost to the country" a reasonable estimate?

[See the back of the book for a hint.]

Exercise 1.8.21. [R][S][Section 1.3][Goal 1.1] The white cliffs of Dover.

In his essay "Season on the Chalk" in the March 12, 2007, issue of *The New Yorker* John McPhee wrote:

> The chalk accumulated at the rate of about one millimetre in a century, and the thickness got past three hundred metres in some thirty-five million years. [**R22**]

Check McPhee's arithmetic.

Exercise 1.8.22. [S][C][Section 1.3][Goal 1.1][Goal 1.4] [Goal 1.5] Social media and internet statistics.

In January 2009 Adam Singer blogged:

> I thought it might be fun to take a step back and look at some interesting/amazing social media, Web 2.0, crowdsourcing and internet statistics. I tried to find stats that are the most up-to-date as possible at the time of publishing this post. [**R23**]

(a) Read that blog entry, choose a few numbers you find interesting, and make sense of them. Are they reasonable? Are they consistent?

(b) Estimate (or research) what those numbers might be now (when you are answering this question.)

(c) We saw this information on the blog: in March 2008, there were 70 million videos on YouTube. It would take 412.3 years to view all of that YouTube content. Thirteen hours of video are uploaded to YouTube every minute. Can you make sense of these numbers? Are they reasonable? Are they consistent?

(d) Can you locate the source of the statistics above, or other sources that confirm them?

Exercise 1.8.23. [S][Section 1.3][Goal 1.1] [Goal 1.3][Goal 1.8] Bottle deposits.

A headline in *The Boston Globe* on July 15, 2010, read: "State panel OKs expansion of nickel deposit to bottled water." At the time, Massachusetts required a 5 cent bottle deposit for all bottles containing carbonated liquids. There was a debate about extending the deposit law to other liquids, including bottled water. In the article you could read that

> The Patrick administration, which supports the bottle bill, has estimated the state would raise about $58 million by allowing the redemption of an additional 1.5 billion containers a year, or about $20 million more than the state earns from the current law, and that municipalities would save as much as $7 million in disposal costs. [**R24**]

(a) Is it reasonable to estimate that 1.5 billion water bottles would be recycled in a year if users paid a nickel deposit on each?

(b) Is $7 million a reasonable estimate of the cost of disposing of 1.5 billion bottles (probably in a landfill) rather than recycling them?

(c) Use the data to estimate the number of water bottles potentially redeemed relative to the number of bottles and cans currently being redeemed. Does the result of the comparison seem reasonable?

(d) The article says the administration estimates that the state will collect $58 million by keeping the deposits paid by the people who don't return the bottles. Use that information to estimate the percentage of bottles that they expect will be recycled.

Note: This bill was defeated in the state legislature.

Exercise 1.8.24. Drivers curb habits as cost of gas soars.

Moved to Extra Exercises at `www.ams.org/bookpages/text-63`.

Exercise 1.8.25. [S][Section 1.3][Goal 1.1] [Goal 1.2][Goal 1.4][Goal 1.5] The Homemade Cafe.

Figure 1.3 appeared on the back of Berkeley California's Homemade Cafe tenth anniversary tee shirt in 1989.

(a) Check that the numbers there make sense.

(b) Assume that the Homemade Cafe is still in business when you are working on this exercise. What numbers would go on this year's tee shirt?

Exercise 1.8.26. [S][Section 1.3][Goal 1.1] [Goal 1.2][Goal 1.3][Goal 1.5] So Many Books, So Little Time.

On Sunday, March 4, 2012, Anthony Doerr calculated in *The Boston Globe* that reading one book a week for 70 years would get you to 3,640 books. Then he wrote:

> If you consider that the Harvard University Library system's collection is counted in the tens of millions, or that a new book of fiction is published every 30 minutes, 3,640 doesn't seem like so many. [**R26**]

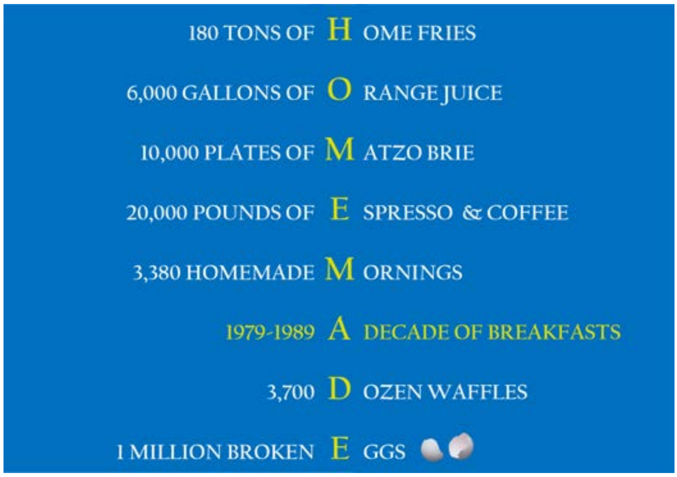

Figure 1.3. The Homemade Cafe [**R25**]

(a) Confirm that 70 years of reading one book per week would amount to 3,640 books read in a lifetime.

(b) How many people reading one book per week during their lifetime would it take to read all the books in the Harvard University Library system?

(c) If you read one book of fiction each week this year, what percent of all the fiction published this year will you have read?

Exercise 1.8.27. Counting car crashes.
Moved to Extra Exercises at `www.ams.org/bookpages/text-63`.

Exercise 1.8.28. [R][S][Goal 1.3][Section 1.4] Do parentheses matter?
What does the Google calculator tell you if you accidentally leave out the parentheses in the computation $12/(2 * 3)$?

Exercise 1.8.29. [U][R][S][Section 1.4][Goal 1.4] Should the U.S. Really Try to Host Another World Cup?

> The proposed budget for the 2010 [Soccer World Cup] games was about \$225 million for stadiums and \$421 million overall. Expenses have far exceeded those numbers. Reported stadium expenses jumped from the planned level of \$225 million to \$2.13 billion, and overall expenses jumped similarly from \$421 million to over \$5 billion. [**R27**]

How many orders of magnitude off were these estimates? That is, how many places were the decimal points away from where they should have been?

Exercise 1.8.30. [U][Section 1.5][Goal 1.5][Goal 1.1] Check our arithmetic, please.
When we found out in Section 1.5 that Baba Brinkman needed just ten seconds to plant a tree, we decided that his claim was possible.

(a) Verify the statement in the section that "… our 100 year estimate should really have been only about two years!"

(b) Verify the next statement: "If it took him a minute per tree rather than 10 seconds he could still have planted a million trees in ten summers."

Exercise 1.8.31. [S][Section 1.6][Goal 1.2][Goal 1.6] [Goal 1.8] The tooth fairy.

How many visits per day does the tooth fairy make in the United States? What's the daily transaction volume (in dollars) in the tooth fairy sector of the economy?

Before you turn to the internet for data, use what you know and some common sense to estimate answers. Write down your assumptions. Only then search the web if you wish.

Exercise 1.8.32. [S][Section 1.6] [Goal 1.2][Goal 1.4][Goal 1.8] Paying for college.

In 2006, the ABC program 20/20 told the story of a couple on Los Angeles who put their children through college by collecting and redeeming soda cans and bottles. Their oldest son went to MIT and their two other children attended California state schools. According to the article, the Garcias collected cans and bottles for 21 years with the goal of saving for their children's college tuition. [**R28**]

Is this possible?

[See the back of the book for a hint.]

Exercise 1.8.33. [S][Section 1.6][Goal 1.2] [Goal 1.4][Goal 1.8] Low flow toilets.

In 1994, a U.S. federal law went into effect that required all new residential toilets to be "low-flow", using just 1.6 gallons of water per flush instead of the five gallons per flush of older toilets.

Estimate how much water a household could save in one year by switching to low flow toilets.

Exercise 1.8.34. [S][W][Section 1.6][Goal 1.1] [Goal 1.5] Vet bills add up.

The inside back cover of the September/October 2008 issue of *BARk* magazine carried an ad for pet insurance asserting that every ten seconds a pet owner faced a $1,000 vet bill.

Is this claim reasonable?

Exercise 1.8.35. [U][Section 1.6][Goal 1.2] [Goal 1.6][Goal 1.8] Americans love animals.

In the November 29, 2009, issue of *The New Yorker* Elizabeth Kolbert wrote in a review of Jonathan Safran Foer's *Eating Animals* that there were 46 million dog-owning households, 38 million with cats and 13 million aquariums with more than 170 million fish.

> Collectively, these creatures cost Americans some forty billion dollars annually. (Seventeen billion goes to food and another twelve billion to veterinary bills.) [**R29**]

Is the twelve billion dollar figure she quotes for veterinary bills consistent with the numbers in the previous problem?

Exercise 1.8.36. [U][Section 1.6][Goal 1.2] [Goal 1.4][Goal 1.5][Goal 1.8] Total carbon footprint.

Use the estimates for the seven tasks discussed in Section 1.6 to rank those tasks in order of *total* daily carbon footprint.

Exercise 1.8.37. [S][Section 1.7][Goal 1.1][Goal 1.5][Goal 1.6] How may internet ads?
An employee from Akamai claimed that

> There are 4.5×10^{12} internet advertisements annually. That's two thousand ads per person per year.

(a) Are the figures for the total number of ads and the number per person consistent?

(b) Do you think two thousand ads per person per year is a good estimate?

Exercise 1.8.38. [R][S][Section 1.7][Goal 1.7] [Goal 1.8] Metric ton.
A *metric ton*, also known as a *tonne*, is 1,000 kilograms.

(a) Is a metric ton a megagram or a gigagram?

(b) How many grams are there in a kilotonne?

(c) How many grams are there in a megatonne?

Exercise 1.8.39. e-reading.
Moved to Extra Exercises at `www.ams.org/bookpages/text-63` .

Exercise 1.8.40. Personal storage.
Moved to Extra Exercises at `www.ams.org/bookpages/text-63` .

Exercise 1.8.41. Backing up the Library of Congress.
Moved to Extra Exercises at `www.ams.org/bookpages/text-63` .

Exercise 1.8.42. [U][Section 1.7][Goal 1.6][Goal 1.7] giga-usa.
The website `www.giga-usa.com/` advertises itself as an

> Extensive collection of 100,000+ ancient and modern quotations, aphorisms, maxims, proverbs, sayings, truisms, mottoes, book excerpts, poems and the like browsable by 6,000+ authors or 3,500+ cross-referenced topics. [**R30**]

Is the website properly named?

Exercise 1.8.43. Data glut.
Moved to Extra Exercises at `www.ams.org/bookpages/text-63` .

Exercise 1.8.44. Zettabytes.
Moved to Extra Exercises at `www.ams.org/bookpages/text-63` .

Exercise 1.8.45. Zettabytes redux.
Moved to Extra Exercises at `www.ams.org/bookpages/text-63` .

Exercise 1.8.46. [W][S][Section 1.3][Goal 1.1] Waiting for the light to change.
In the Pooch Cafe comic strip on August 27, 2012, Poncho the dog is sitting in the car with his master Chazz. He says, "Did you know the average person spends six months of their life waiting at red lights?" (You can see the strip at `www.gocomics.com/poochcafe/2012/08/27`)

What do you think of Poncho's estimation skills?

Exercise 1.8.47. Killer cats.

Moved to Extra Exercises at `www.ams.org/bookpages/text-63`.

Exercise 1.8.48. [S][Section 1.5][Goal 1.1][Goal 1.6] Viagra, anyone?

A 2014 Viagra ad on TV stated that more than 20 million men already use Viagra. Use the 2018 U.S. population pyramid in Figure 1.4 to argue whether or not this claim seems reasonable. Be explicit about any assumptions you make about the age groups of men who might typically use this drug.

Does using the 2018 population pyramid rather than one from 2014 affect your conclusion?

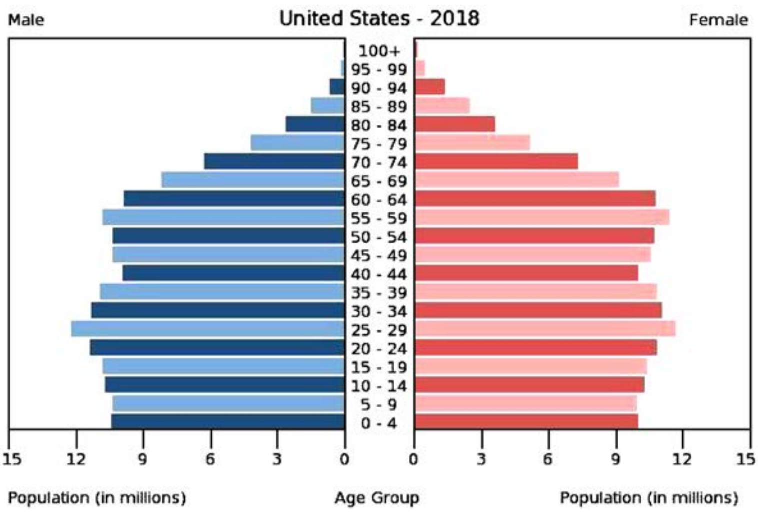

Figure 1.4. U.S. Population pyramid (2018) [**R31**]

Exercise 1.8.49. [U][Section 1.1][Goal 1.1][Goal 1.2] Lots of olives?

On August 23, 2014, the Associated Press reported that in the 1980's Robert Crandall, the CEO of American Airlines decided that offering one less olive in every salad would save $40,000 a year. [**R32**]

(a) About how many olives could Crandall buy for $40,000?

(b) Is your answer to the previous question in the same ballpark as the number of American Airlines passengers in 1980?

Exercise 1.8.50. [U][C][Section 1.3][Goal 1.1] [Goal 1.2][Goal 1.5][Goal 1.8] Nuclear bombs.

"Kiloton" and "megaton" are terms you commonly hear when nuclear bombs are being discussed. In that context the "ton" refers not to 2,000 pounds, but to the explosive yield of a ton of TNT.

(a) What was the explosive yield of the only (two) atomic bombs ever used in war?

(b) How does the explosive yield of the hydrogen bombs in the current arsenals of the United States and Russia (and other countries) compare to that of those first atomic bombs?

(c) Estimate the destructive power of the world's current stockpile of nuclear weapons, in terms easier to grasp than kilotons or megatons or gigatons or ....

Review exercises.

Exercise 1.8.51. [A] Do each of these calculations by counting zeroes. Use the Google or Bing calculator (or another calculator) to check your answers.

(a) One million times one billion.

(b) Four hundred times three thousand.

(c) Two billion divided by two hundred.

(d) One-tenth times five thousand.

(e) Four thousand divided by two hundred.

(f) Twelve thousand times two hundred.

(g) $10,000 \times \frac{2}{1,000}$.

(h) $450,000 \times 100$.

(i) $\frac{50,000}{200}$.

(j) $\frac{4,000,000,000,000}{2,000,000}$.

Exercise 1.8.52. [A] Use rounding to estimate each of these quantities. Then check your answers.

(a) The number of feet in ten miles.

(b) The number of minutes in a week.

(c) The number of inches in a mile.

(d) The number of yards in a mile.

(e) The number of seconds in a month.

Exercise 1.8.53. [A] Answer each of these questions without using pencil and paper (or a calculator).

(a) There are twelve cans of soda in a case. If I buy five cases of soda, how many cans will I have in total?

(b) A case of bottled water contains 24 bottles. About how many bottles are there in four cases?

(c) At the market my cart contains three bags of cereal at $2.19 each, a gallon of milk for $2.99, a pound of potatoes that costs $3.50 and a $3.99 bag of apples. Will my total be more than twenty dollars?

(d) At our local deli, a bagel costs $1.19. How much would five bagels cost? If the deli offers six bagels for $6.99, is that a better deal?

(e) It costs $2.50 to ride the New York City subway. A seven-day unlimited pass costs $29. How many rides in a seven day period should you take before it's a better deal to buy the pass?

(f) I have a $100 gift card for a department store. Can I buy a pair of shoes for $19.99, two shirts that are $12.59 each and three pairs of pants that each cost $20.50?

Exercise 1.8.54. [A] Convert the following measurements:

(a) One kilometer into meters.

(b) One megameter into kilometers.

(c) One terabyte into megabytes.

(d) Three megawatts into kilowatts.

(e) Five gigabytes into kilobytes.

(f) One thousand kilograms into megagrams.

Exercises added for the second edition.

Exercise 1.8.55. [S][Section 1.1][Goal 1.1][Goal 1.2][Goal 1.4][Goal 1.6] Chicken salad all around.

On May Day 2019 *Bloomberg* reported from the Milken Institute Global Conference that

> In Beverly Hills, the chicken Caesar salad costs $25.95. Stephen Schwarzman, with a net worth of $14.3 billion, could buy one for all 329 million people in the U.S. today — and then do so again tomorrow for 222 million of them. [**R33**]

(a) How would you verify this estimate using just simple arithmetic, without a calculator?

(b) Check the estimate with a calculator.

Exercise 1.8.56. [U][S][Section 1.2][Goal 1.1][Goal 1.6][Goal 1.8] Jeff Bezos is really rich.

In October 2018 the website QUARTZ*at*WORK posted a story about Amazon's plan to pay its 250,000 full time and 100,000 part time workers $15 an hour. Amazon claims that their lowest paid employee would make about $30,000 a year.

The article then notes that

> …in the past 12 months, [founder Jeff Bezos's] net worth increased by $82.6 billion. …Presuming his wealth creation continues at a similar pace, Bezos will "make" the annual salary of one of Amazon's newly minted $15/hour employees every 11.5 seconds. [**R34**]

(a) Check that at $15/hour "Amazon's lowest-paid full-time employee in the U.S. will make around $30,000 a year."

(b) Compare what Amazon distributes yearly in wages for those employees to the projected increase in Bezos's wealth.

(c) Check the claim that Bezos will collect \$30,000 every 11.5 seconds is in the right ballpark.

Exercise 1.8.57. [U][S][Section 1.1][Goal 1.2] A million pitches.

In *The Boston Globe* on June 16, 2015, Stan Grossfeld wrote that

> In the 105-year history of Fenway, hitters have faced more than a million pitches. [**R35**]

Is that million pitches a reasonable estimate?
[See the back of the book for a hint.]

Exercise 1.8.58. [S][Section 1.3][Goal 1.1][Goal 1.2] [Goal 1.4] Dropping out.

On the website DoSomething.org you can read that

> Every year, over 1.2 million students drop out of high school in the United States alone. That's a student every 26 seconds — or 7,000 a day. [**R36**]

Are the three numbers in this quotation consistent? If not, can you explain the discrepancy?

Exercise 1.8.59. [S][Section 1.7][Goal 1.6][Goal 1.2][Goal 1.7] Gone phishin'

On January 13, 2019, *The Boston Globe* reported on a small company helping to counter internet phishing scams.

> [T]he data it produced on the [recent Netflix] attack were among 532,765,897 million fields of data it provided last month alone. [**R37**]

The "fields" referred to are the boxes you fill in on a form in your browser. Figure 1.5 shows a Netflix registration form with two fields, one for an email address, one for a password. Later in the process you would be asked for your name and your credit card information. We estimate that the average length of the fields is at least 10 characters, which you can think of as at least 10 bytes.

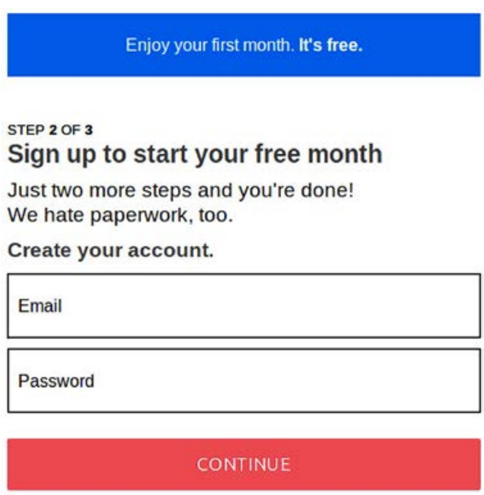

Figure 1.5. Netflix account creation form

(a) What is “phishing”?

(b) Write the number in this quotation as a number of bytes, using the appropriate metric prefix and an appropriate number of significant digits.

(c) Estimate the number of Netflix requests to open an account or order a movie it would take to generate that much data.

(d) Argue convincingly that the number is much too large to have been correctly reported.

Exercise 1.8.60. [S][Section 1.2][Goal 1.1][Goal 1.2][Goal 1.6] Counting galaxies.

Until recently astrophysicists estimated that the universe contained 100 to 200 billion galaxies. Then an international team proposed a minimum tenfold increase to 1 to 2 trillion. The AP report on the study concluded with this observation from Professor Conselice of Nottingham University, the lead researcher on the team, on how hard it is to imagine such large numbers:

> 2 trillion is equivalent to the number of seconds in 1,000 average lifetimes. [**R38**]

(a) Is the new estimate really a minimum tenfold increase?

(b) Is Conselice’s count of seconds right?

Exercise 1.8.61. [U][S][Section 1.2][Goal 1.2][Goal 1.6] So many bitcoins, so many Benjamins.

In his *The New York Times* op-ed on January 29, 2018, Paul Krugman wrote:

> Like Bitcoins, $100 bills aren’t much use for ordinary transactions: Most shops won’t accept them. But “Benjamins” are popular with thieves, drug dealers and tax evaders. And while most of us can go years without seeing a $100 bill, there are a lot of those bills out there — more than a trillion dollars’ worth, accounting for 78 percent of the value of U.S. currency in circulation. [**R39**]

(a) How many Benjamins are there in circulation?

(b) Use the data in this exercise to estimate the value of U.S. currency in circulation.

(c) Confirm that estimate with a web search.

(d) Why is the $100 bill a “Benjamin”?

Exercise 1.8.62. [S][Section 1.1][Goal 1.6] Don’t answer the phone.

In April 2019, *The New York Times* reported that

> The seemingly endless stream of robocalls reached a new monthly high of 5.23 billion nationwide in March, according to the call-blocking service YouMail. [**R40**]

Write a short paragraph in which you convert the 5.23 billion robocalls into a number that you can grasp. Then discuss why you think it’s too small, too large, or just right.

Exercise 1.8.63. [U][S][Section 1.1][Goal 1.1][Goal 1.2][Goal 1.6] One hundred billion bottles.

On October 29, 2019, the Portland Maine *PressHerald* featured an Associated Press report that said

> Every year, an estimated 100 billion plastic bottles are produced in the U.S. [**R41**]

(a) Show that this claim is about one bottle per person per day. You can do that with common knowledge and a little elementary arithmetic. No need for a calculator or the internet.

(b) Write a sentence or two about whether you think this claim is correct.