

Contents

Preface to the Second Edition	xi
--------------------------------------	----

Preface to the First Edition	xiii
-------------------------------------	------

1 Calculating on the Back of an Envelope	1
---	---

In this first chapter we learn how to think about questions that need only good enough answers. We find those answers with quick estimates that start with reasonable assumptions and information you have at your fingertips. To make the arithmetic easy we round numbers drastically and count zeroes when we have to multiply.

1.1 Hailing a ride	2
1.2 How many seconds?	4
1.3 Heartbeats	5
1.4 Calculators	6
1.5 Millions of trees?	7
1.6 Carbon footprints	8
1.7 Kilo, mega, giga	9
1.8 Exercises	10

2 Units and Unit Conversions	27
-------------------------------------	----

In real life there are few naked numbers. Numbers usually measure something like cost, population, time, speed, distance, weight, energy or power. Often what's measured is a rate, like miles per hour, gallons per mile, miles per gallon, dollars per gallon, dollars per euro or centimeters per inch.

2.1 Rate times time equals distance	27
2.2 The MPG illusion	29
2.3 Converting currency	31
2.4 Unit pricing and crime rates	33
2.5 The metric system	34
2.6 Working on the railroad	36
2.7 Scientific notation, milli and micro	37
2.8 Carpeting and paint	39
2.9 Exercises	41

3 Percentages, Sales Tax and Discounts 65

The focus of this chapter is the study of relative change, often expressed as a percent. We augment an often much needed review in two ways — stressing quick paperless estimation for approximate answers and, for precision, a new technique: multiplying by $1+$ (percent change).

3.1	The federal budget	65
3.2	Red Sox ticket prices	68
3.3	The $1+$ trick	68
3.4	Exploiting the $1+$ trick	70
3.5	Large and small percentages	71
3.6	Percentage points	73
3.7	Percentiles	73
3.8	Exercises	74

4 Inflation 101

We mine the internet for data about inflation and use the $1+$ technique from Chapter 3 to understand that data.

4.1	Red Sox ticket prices	101
4.2	Inflation is a rate	102
4.3	The Consumer Price Index	104
4.4	More than 100 %	104
4.5	How much is your raise worth?	105
4.6	The minimum wage	105
4.7	Inflation history	107
4.8	Exercises	108

5 Average Values 117

We start by remembering that to compute an average you add the values and divide by the count. We quickly move on to weighted averages, which are more common and more useful. They're a little harder to understand, but worth the effort. They help explain some interesting apparent paradoxes.

5.1	Average test score	117
5.2	Grade point average	118
5.3	Improving averages	119
5.4	The Consumer Price Index	120
5.5	New car prices fall ...	122
5.6	An averaging paradox	123
5.7	Exercises	123

6 Income Distribution — Spreadsheets, Charts and Statistics 131

This chapter covers a lot of ground — two new kinds of average (median and mode) and ways to understand numbers when they come in large quantities rather than just a few at a time: bar charts, histograms, percentiles and the bell curve. To do that we introduce spreadsheets as a tool.

Contents	vii
6.1 Salaries at Wing Aero	132
6.2 What if?	136
6.3 Using software	136
6.4 Median	137
6.5 Bar charts	138
6.6 Pie charts	141
6.7 Histograms	142
6.8 Mean, median, mode	144
6.9 Computing averages from histograms	145
6.10 The bell curve	146
6.11 Margin of error	149
6.12 Exercises	150
7 Electricity Bills and Income Taxes — Linear Functions	169
We use an electricity bill as a hook on which to hang an introduction to functions in general and linear functions in particular, in algebra and in spreadsheets. Then we apply what we've learned to study taxes — sales, income and Social Security. You'll also find here a general discussion of energy and power.	
7.1 Rates	169
7.2 Reading your electricity bill	170
7.3 Linear functions	172
7.4 Linear functions in a spreadsheet	173
7.5 Which truck to rent?	177
7.6 Energy and power	178
7.7 Federal payroll taxes	180
7.8 Exercises	185
8 Climate Change — Linear Models	199
Complicated physical and social phenomena rarely behave linearly, but sometimes data points lie close to a straight line. When that happens you can use a spreadsheet to construct a linear approximation. Sometimes that's useful and informative. Sometimes it's misleading. Common sense can help you understand which.	
8.1 Climate change	199
8.2 The greenhouse effect	203
8.3 How good is the linear model?	205
8.4 Regression nonsense	206
8.5 Exercises	209
9 Compound Interest — Exponential Growth	217
In this chapter we explore how investments and populations grow and how radioactivity decays — exponentially.	
9.1 Money earns money	217
9.2 Exploring exponential growth with a spreadsheet	219
9.3 Depreciation	221
9.4 Doubling times and half-lives	222
9.5 Exponential models	225

9.6	“Exponentially”	227
9.7	Exercises	228
10	Borrowing and Saving	243
	When you borrow money — on your credit card, for tuition, for a mortgage — you pay it back in installments. Otherwise what you owe would grow exponentially. In this chapter we explore the mathematics that describes paying off your debt.	
10.1	Debit and credit cards	243
10.2	Can you afford a mortgage?	247
10.3	Saving for college or retirement	249
10.4	Effective interest rate	250
10.5	Instantaneous compounding	251
10.6	Exercises	251
11	Probability — Counting, Betting, Insurance	257
	Pierre de Fermat and Blaise Pascal invented the mathematics of probability to answer gambling questions posed by a French nobleman in the seventeenth century. We follow history by starting this chapter with simple examples involving cards and dice. Then we discuss raffles and lotteries, fair payoffs and the house advantage, insurance, and risks where quantitative reasoning doesn’t help at all.	
11.1	Equally likely	257
11.2	Odds	258
11.3	Raffles	259
11.4	State lotteries	260
11.5	The house advantage	263
11.6	One-time events	265
11.7	Insurance	266
11.8	Sometimes the numbers don’t help at all	267
11.9	Exercises	268
12	Break the Bank — Independent Events	275
	Unlikely things happen — just rarely! Here we calculate probabilities for combinations like runs of heads and tails. Then we think about luck and coincidences.	
12.1	A coin and a die	275
12.2	Repeated coin flips	277
12.3	Double your bet?	279
12.4	Cancer clusters	280
12.5	The hundred year flood	281
12.6	Improbable things happen all the time	283
12.7	Exercises	284
13	How Good Is That Test?	291
	In Chapter 12 we looked at probabilities of independent events — things that had nothing to do with one another. Here we think about probabilities in situations where we expect to see connections, such as in screening tests for diseases or DNA evidence for guilt in a criminal trial.	

Contents	ix
13.1 UMass Boston enrollment	291
13.2 False positives and false negatives	293
13.3 Screening for a rare disease	295
13.4 Trisomy 18	296
13.5 The prosecutor's fallacy	298
13.6 The boy who cried "Wolf"	299
13.7 Exercises	300
Hints	307
References	313
Index	337

Contents

Preface to the Second Edition	xi
--------------------------------------	----

Preface to the First Edition	xiii
-------------------------------------	------

1 Calculating on the Back of an Envelope	1
---	---

In this first chapter we learn how to think about questions that need only good enough answers. We find those answers with quick estimates that start with reasonable assumptions and information you have at your fingertips. To make the arithmetic easy we round numbers drastically and count zeroes when we have to multiply.

1.1 Hailing a ride	2
1.2 How many seconds?	4
1.3 Heartbeats	5
1.4 Calculators	6
1.5 Millions of trees?	7
1.6 Carbon footprints	8
1.7 Kilo, mega, giga	9
1.8 Exercises	10

2 Units and Unit Conversions	27
-------------------------------------	----

In real life there are few naked numbers. Numbers usually measure something like cost, population, time, speed, distance, weight, energy or power. Often what's measured is a rate, like miles per hour, gallons per mile, miles per gallon, dollars per gallon, dollars per euro or centimeters per inch.

2.1 Rate times time equals distance	27
2.2 The MPG illusion	29
2.3 Converting currency	31
2.4 Unit pricing and crime rates	33
2.5 The metric system	34
2.6 Working on the railroad	36
2.7 Scientific notation, milli and micro	37
2.8 Carpeting and paint	39
2.9 Exercises	41

3 Percentages, Sales Tax and Discounts 65

The focus of this chapter is the study of relative change, often expressed as a percent. We augment an often much needed review in two ways — stressing quick paperless estimation for approximate answers and, for precision, a new technique: multiplying by $1+$ (percent change).

3.1	The federal budget	65
3.2	Red Sox ticket prices	68
3.3	The $1+$ trick	68
3.4	Exploiting the $1+$ trick	70
3.5	Large and small percentages	71
3.6	Percentage points	73
3.7	Percentiles	73
3.8	Exercises	74

4 Inflation 101

We mine the internet for data about inflation and use the $1+$ technique from Chapter 3 to understand that data.

4.1	Red Sox ticket prices	101
4.2	Inflation is a rate	102
4.3	The Consumer Price Index	104
4.4	More than 100 %	104
4.5	How much is your raise worth?	105
4.6	The minimum wage	105
4.7	Inflation history	107
4.8	Exercises	108

5 Average Values 117

We start by remembering that to compute an average you add the values and divide by the count. We quickly move on to weighted averages, which are more common and more useful. They're a little harder to understand, but worth the effort. They help explain some interesting apparent paradoxes.

5.1	Average test score	117
5.2	Grade point average	118
5.3	Improving averages	119
5.4	The Consumer Price Index	120
5.5	New car prices fall ...	122
5.6	An averaging paradox	123
5.7	Exercises	123

6 Income Distribution — Spreadsheets, Charts and Statistics 131

This chapter covers a lot of ground — two new kinds of average (median and mode) and ways to understand numbers when they come in large quantities rather than just a few at a time: bar charts, histograms, percentiles and the bell curve. To do that we introduce spreadsheets as a tool.

Contents	vii
6.1 Salaries at Wing Aero	132
6.2 What if?	136
6.3 Using software	136
6.4 Median	137
6.5 Bar charts	138
6.6 Pie charts	141
6.7 Histograms	142
6.8 Mean, median, mode	144
6.9 Computing averages from histograms	145
6.10 The bell curve	146
6.11 Margin of error	149
6.12 Exercises	150
7 Electricity Bills and Income Taxes — Linear Functions	169
<p>We use an electricity bill as a hook on which to hang an introduction to functions in general and linear functions in particular, in algebra and in spreadsheets. Then we apply what we've learned to study taxes — sales, income and Social Security. You'll also find here a general discussion of energy and power.</p>	
7.1 Rates	169
7.2 Reading your electricity bill	170
7.3 Linear functions	172
7.4 Linear functions in a spreadsheet	173
7.5 Which truck to rent?	177
7.6 Energy and power	178
7.7 Federal payroll taxes	180
7.8 Exercises	185
8 Climate Change — Linear Models	199
<p>Complicated physical and social phenomena rarely behave linearly, but sometimes data points lie close to a straight line. When that happens you can use a spreadsheet to construct a linear approximation. Sometimes that's useful and informative. Sometimes it's misleading. Common sense can help you understand which.</p>	
8.1 Climate change	199
8.2 The greenhouse effect	203
8.3 How good is the linear model?	205
8.4 Regression nonsense	206
8.5 Exercises	209
9 Compound Interest — Exponential Growth	217
<p>In this chapter we explore how investments and populations grow and how radioactivity decays — exponentially.</p>	
9.1 Money earns money	217
9.2 Exploring exponential growth with a spreadsheet	219
9.3 Depreciation	221
9.4 Doubling times and half-lives	222
9.5 Exponential models	225

9.6	“Exponentially”	227
9.7	Exercises	228
10	Borrowing and Saving	243
	When you borrow money — on your credit card, for tuition, for a mortgage — you pay it back in installments. Otherwise what you owe would grow exponentially. In this chapter we explore the mathematics that describes paying off your debt.	
10.1	Debit and credit cards	243
10.2	Can you afford a mortgage?	247
10.3	Saving for college or retirement	249
10.4	Effective interest rate	250
10.5	Instantaneous compounding	251
10.6	Exercises	251
11	Probability — Counting, Betting, Insurance	257
	Pierre de Fermat and Blaise Pascal invented the mathematics of probability to answer gambling questions posed by a French nobleman in the seventeenth century. We follow history by starting this chapter with simple examples involving cards and dice. Then we discuss raffles and lotteries, fair payoffs and the house advantage, insurance, and risks where quantitative reasoning doesn’t help at all.	
11.1	Equally likely	257
11.2	Odds	258
11.3	Raffles	259
11.4	State lotteries	260
11.5	The house advantage	263
11.6	One-time events	265
11.7	Insurance	266
11.8	Sometimes the numbers don’t help at all	267
11.9	Exercises	268
12	Break the Bank — Independent Events	275
	Unlikely things happen — just rarely! Here we calculate probabilities for combinations like runs of heads and tails. Then we think about luck and coincidences.	
12.1	A coin and a die	275
12.2	Repeated coin flips	277
12.3	Double your bet?	279
12.4	Cancer clusters	280
12.5	The hundred year flood	281
12.6	Improbable things happen all the time	283
12.7	Exercises	284
13	How Good Is That Test?	291
	In Chapter 12 we looked at probabilities of independent events — things that had nothing to do with one another. Here we think about probabilities in situations where we expect to see connections, such as in screening tests for diseases or DNA evidence for guilt in a criminal trial.	

Contents	ix
13.1 UMass Boston enrollment	291
13.2 False positives and false negatives	293
13.3 Screening for a rare disease	295
13.4 Trisomy 18	296
13.5 The prosecutor's fallacy	298
13.6 The boy who cried "Wolf"	299
13.7 Exercises	300
Hints	307
References	313
Index	337