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In real life there are few naked numbers. Numbers usually measure something like cost, population, time, speed, distance, weight, energy or power. Often what's measured is a rate, like miles per hour, gallons per mile, miles per gallon, dollars per gallon, dollars per euro or centimeters per inch.

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The focus of this chapter is the study of relative change, often expressed as a percent. We augment an often much needed review in two ways — stressing quick paperless estimation for approximate answers and, for precision, a new technique: multiplying by $1+$ (percent change).

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