Preface

Linear algebra is a central course in undergraduate mathematics programs. It prepares students for the mathematical demands of courses in physics, chemistry, computer science, and engineering as well as for the study of mathematical models in biology, economics, data science, and other quantitative disciplines. At the same time it serves as a transition from the mechanical manipulations of algebra and calculus to more theoretical upper-level mathematics courses.

Expectations

Students should enter the course with the equivalent of two or three semesters of college-level mathematics (typically calculus with perhaps an introduction to differential equations and some work with vectors in the plane and three-dimensional Euclidean space). They will leave with an understanding of the basic results of linear algebra and an appreciation of the beauty and utility of mathematics. They will also be fortified with a degree of mathematical maturity required for subsequent courses in abstract algebra, real analysis, and elementary topology. Students who have additional background in dealing with the mechanical operations of vectors and matrices will benefit from seeing this material placed in a more general context.

Objectives

This textbook is designed to resolve the conflict between the abstractions of linear algebra and the needs and abilities of the students who may have dealt only briefly with the theoretical aspects of previous mathematics courses.

Students will repeatedly encounter discussions of the advantages of dealing with the general theory. The text points out how the similarities among numerous examples can be understood and clarified under a unifying concept and how a general theorem reduces the need for computation. At the same time, a careful attempt is made to distinguish between the abstract notions and the particular examples to which they apply. Applications of mathematics are integrated into the text to reinforce the theoretical material and to illustrate its usefulness.

Another theme of this book, and perhaps the most important one, is the recognition that many students will at first feel uncomfortable or at least unfamiliar with the theoretical nature inherent in many of the topics of linear algebra. There are numerous discussions of the logical structure of proofs, the need to translate terminology into notation, and suggestions about efficient ways to discover a proof. Frequent connections are made with familiar properties of algebra, geometry, and calculus. Although complex numbers make a brief appearance when needed to discuss the roots of the characteristic polynomial of a matrix, real numbers are consistently used as scalars. This strengthens the ties between linear algebra and geometry while setting the stage for the introduction of other fields of scalars in an advanced course in linear algebra.

This book combines the many simple and beautiful results of elementary linear algebra with some powerful computational techniques to demonstrate that mathematics is enlightening, powerful, and practical.

Organization

The first chapter introduces the fundamental concepts of vector spaces. Mathematical notation and proof techniques are carefully discussed in the course of deriving the basic algebraic properties of vector spaces. This work is immediately tied to three families of concrete examples. With this background, students can appreciate the matrix reduction technique discussed in the second chapter as a powerful tool that will be applied in the subsequent study of vector spaces and linear transformations.

Chapter 3 continues the study of vector spaces, culminating in results about the dimension of a vector space. Chapter 4 then introduces the structures of inner products and norms to quantify some of the geometric concepts discussed previously.

Chapter 5 deals with matrix multiplication and inverses. An introduction to Markov chains provides a fascinating opportunity to examine some nontrivial applications of these results. Chapter 6 relates this material to the general concept of linear functions between vector spaces.

Chapter 7 begins with a careful presentation of mathematical induction. An inductive definition leads to the straightforward although somewhat computational presentation of the theory of determinants. Enough guidance is given so the student will not become lost or begin to feel that everything is as complicated as the most intricate detail. The final chapter on eigenvalues and eigenvectors ties together many of the major topics of the course.

Pedagogical Features

Several distinctive features assist students as they study for the course, work on assignments, and review for examinations. Instructors will also find these features helpful in preparing lectures and assignments.

Mathematical Strategy Sessions. These discussions will nurture students in their understanding of definitions, use of proof techniques, and familiarity with mathematical notation.

Crossroads. Students will appreciate these indications of how topics fit together, where ideas will be used in future sections, how a concept unifies material from previous courses, and suggestions for related readings and explorations.

Quick Examples. Here are concise examples and typical problems making direct use of the material in the text. Students can expect to find examples similar to these on homework assignments and examinations.

Exercises. Each section contains a variety of exercises. Some are routine drill problems, but the emphasis is on honest problems (at a range of levels of difficulty) that illustrate the concepts presented in the section or their development in relation to other topics. Occasional open-ended problems encourage students to develop their mathematical creativity. Exercises often set the stage for topics appearing in later sections.

Hints and answers to approximately one-third of the exercises appear at the end of the book. The problem numbers for exercises with hints are flagged with H; those for which answers are provided are flagged with A. Students can check their work against the answers provided for this representative sample of the exercises. Complete solutions are presented for some exercises that demonstrate important concepts or are particularly instructive. These have problem numbers flagged with S.

Projects. Each chapter contains suggested projects for student investigation. Some projects are in the form of a guided tour with interesting side issues for students to think about. Others are mere sketches with suggested references and an invitation to explore the resources available in the mathematics section of the library. These projects may be used for independent investigation or for a group of students to work on cooperatively. Students who want to focus on a portion of one of these projects will nevertheless see the larger context for their work. These sections may also be used as reading assignments to make students aware of the role of linear algebra in mathematics and its application to other disciplines. This may plant some seeds for more ambitious investigations in a senior seminar or undergraduate research project.

References for related material. Additional references are provided for students interested in pursuing topics related to the course or previewing more advanced topics.

Chapter summaries. A concise overview of the accomplishments of each chapter will reinforce the theme of placing details in a larger context. An outline of concepts reorganizes the material from the chapter as computation, theory, or application. This will aid students in studying for exams without converting the course into a list of terms and techniques to be memorized.

Chapter review exercises. Additional sets of problems recall the highlights of each chapter. These exercises are again at a variety of levels. They often bring together topics from more than one section. This encourages students to see the development of the material from a broad perspective rather than filing each section in a separate mental compartment.

Acknowledgments

This book began as an experiment in on-site publishing. The original version was written, edited, typeset, and tested at Albion College. It has undergone numerous revisions in response to helpful comments from many people. George Duda, editor at HarperCollins, has contributed wisdom and guidance in adding magic to transform the original manuscript into a distinctive textbook. It was a pleasure to work with Shirley Cicero, Carol Zombo, and James Harris. David Mason did an amazing job of removing stylistic and grammatical irregularities from the manuscript. The reviewers, listed below, made valuable suggestions for exercises, examples, clarifications, and other improvements.

John F. Cavalier	West Virginia Institute of Technology	Montgomery, WV
Stuart Goff	Keene State College	Keene, NH
Sydney Graham	Michigan Technological University	Houghton, MI
Benny P. Lo	Ohlone College	Fremont, CA
Roger Lutz	Western New Mexico University	Silver City, NM
Andy Magid	University of Oklahoma	Norman, OK
Ana Mantilla	University of Arizona	Tuscon, AZ
Donald Passman	University of Wisconsin	Madison, WI
Patrick Stewart	Dalhousie University	Halifax, NS

The Editorial Board of the Mathematical Association of America provided extensive comments and suggestions that have been incorporated into the second edition of this textbook.

Special thanks to Sylvia Hobart for her help in converting the TEX files to LATEX.

Colleagues Paul Anderson, Ruth Favro, John Fink, Raymond Greenwell, William Higgins, Lisa Holden, Norman Loomer, and Eric Wilson checked over the solutions to the exercises. The following students, professors, and other readers have made suggestions for improvements in earlier versions of this book. Their help is gratefully acknowledged.

Albion College: Geoffrey Armstrong, Thomas Basso, Charmian Bultema, Josh Cassada, Sascha Chin, Matthew Chittle, Sue Colton, Douglas Copley, Kenneth Davis, Michelle Diener, Erik Eid, Paul Enns, Penny Eveningred, Rachelle Fichtner, Joe Gibson, Julie Hamrick, Jafar Hasan, John Hill, Diane Hines, David Holden, Jacob Hooker, Fred Horein, Michelle Hribar, Marco Jeromin, Michael Juchno, Michael Kidder, Phillip Koppers, David Kutcipal, Adam LaPratt, Kevin Lepard, Martin Ludington, Vincent Magnotta, Steven Malinak, John Mark Meldrim, Pat Messer, Joseph Meyers, Renée Miller, Debra Neumeyer, Martha O'Kennon, David Oliver, Andrea Ondracek, Amanda Parke, Gregory Parker, Michael Parr, Sarah Paukstis, John Peternel, Polly Reeder, Christopher Richardson, Anna Rizzi, Jackie Salyer, Karl Schwartz, Bryan Sladek, Jon Stewart, Ronald Targosz, Alex Tashian, Phil Thorson, Linda Valachovic, Greg Wallender, Robert Wells, John Wenzel, Aaron Werbling, and Charles Yun

Indiana State University: Steven Edwards

Ripon College: Norm Loomer

Spring Arbor University: Charles Carey, Katie Marema, and Benjamin Rohrer

University of Copenhagen: Trine E. Andersen, Christian Berg, Ask Jakobsen, Anders Bo Nielsen, and Sune Olander Rasmussen

University of Wisconsin: Lawrence Levy and Donald Passman

Wittenberg University: William Higgins, Al Stickney, and Eric Wilson

Your comments about any aspect of this book are welcome. The names will be acknowledged of those whose comments or suggestions are incorporated in future editions. You may contact the author at the address below.

Robert Messer
Department of Mathematics and Computer Science
Albion College
Albion, Michigan 49224-5013
e-mail: ram@albion.edu