Preface

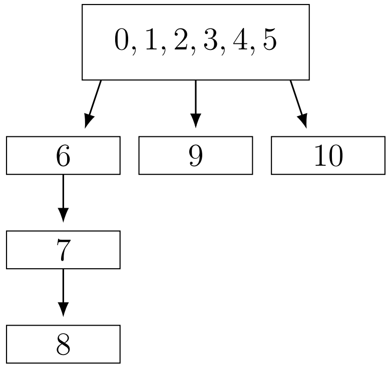
This book is written for use in an introductory undergraduate real analysis course. Its purpose is to acquaint students with some of the central ideas in real analysis and a variety of their applications. In this book, these applications fall mostly into two main streams: single-variable calculus and discrete dynamical systems. My aim is to help students get to know real analysis both as the foundation of calculus and as much more besides. I hope that the book will equip and inspire students to pursue further study in a range of topics, including functional analysis and integration, differential calculus in higher dimensions, probability and statistics, complex analysis, and dynamical systems.

I feel that a first exposure to real analysis should approach the topic with enough generality that students see the wide reach of the core concepts and do not make the mistake of thinking that distinct settings are more different than they really are. Accordingly, in addition to undertaking much of the development in general metric spaces, I have sought to make substantial use not only of the space $\mathbb{R}$ but of some other important metric spaces as well. In particular, the space $C([a, b], \mathbb{R})$ of continuous real-valued functions on a compact interval is introduced in Chapter 4 and plays an important role in Theorems 7.4.4 and 8.4.1. The sequence space $\mathcal{S}_n$ on n symbols is introduced in Chapter 3 and plays a pivotal role in the study of chaos in Chapter 9.

At the same time, I have simplified some material in an attempt to keep the main ideas as vivid as possible. For example, I only consider power series that are real and centered at zero; I state somewhat simplified versions of the root and ratio tests; and for integration I discuss only the Riemann theory, and only in one dimension.

The book is intended for an upper-level undergraduate audience. I use material from calculus and (to a lesser extent) linear algebra as a source of motivation and examples. Although I assume that readers have some initial experience with rigorous mathematics, I include some explicit coaching and advice on understanding, finding, and writing proofs, especially in the early chapters.

There is more in the book than fits comfortably into a single-semester course. When I teach this material I move relatively slowly through Chapters 1–4; by the time we reach later chapters, we can move a bit faster because students are accustomed to how the basic arguments go (and, in Chapters 6 and 7, already know many of the theorems). The first seven chapters constitute a “core” analysis course, with the fundamental theorem of calculus and existence and uniqueness of solutions of ordinary differential equations as a climax. The remaining chapters are wholly independent of each other and can be included according to the preference of a class and its instructor. If a class wishes to focus more on dynamics than on calculus, Chapter 9 or Chapter 10 can be started any time after Chapter 5. A thorough use of the entire book can also be



part of a two-semester course, especially if the class is relatively new to abstract mathematics at the outset and accordingly will benefit from digesting the early chapters at a relaxed pace. The diagram shows the dependency of the chapters.

The “reading questions” that follow each section are problems designed to help students absorb the main ideas. Many of the reading questions are very simple, requiring no more than the recollection of a definition or example from the text; some are more challenging but seem to me well-suited to accompany digestion and discussion of the reading. When I teach from this book, I use part of each class meeting for a group discussion of the reading questions. Each chapter concludes with exercises that range in difficulty from routine to challenging; when I teach this course, I assign weekly homework assignments from the exercises. Some of the exercises have boldfaced titles describing their content; I have identified in this way exercises that address familiar topics that are not otherwise discussed in the book (such as the second derivative test and the definition of the sine and cosine functions). A class might choose to make exercises like this, especially, subjects for group discussion.

While the material in this book is mostly standard, my debts to certain other expository works are notable. I myself first learned analysis from the books by Spivak [**18**] and Rosenlicht [**15**]. I found much in Abbott’s book [**1**] inspiring, especially the early introduction of sequences and the approach to Riemann sums. My thinking about teaching introductory dynamical systems was strongly influenced by Alligood, Sauer, and Yorke’s book [**2**], in addition to the classic textbooks [**6**] and [**7**] by Devaney. Much of the material in Chapter 1 is influenced by the unique book [**3**] of my colleague Béla Bajnok. A handful of other places where I rely on other sources are cited in the text.

Students who have used drafts of this book have provided me with many astute and useful comments. Particular thanks are due to Adrian Navarro, Wyatt Hathaway, Yiran Mao, Peter Francis, Ashley Gaffey, Kyle Beatty, and Ben Nagle. The members of the editorial board of the AMS/MAA Textbook series have given me many valuable suggestions, and I am most grateful to them. I thank my colleague Beth Campbell Hetrick for trying out an early version of this book with her students. I owe much to Aaron Hoffman for our many wide-ranging conversations about teaching and mathematics.

Finally, I express my deepest appreciation to my student Matthew Torrence who, when using a draft of this text in class, offered his assistance in improving the figures. Matt re-worked the figures to make them more readable and attractive, and has greatly enhanced this book.