

# Contents

<b>Preface</b>	xi
<b>0 Where We're Starting and Where We're Going</b>	1
<b>1 Essential Tools</b>	5
1.1 Sets and statements	5
1.2 Functions	9
1.3 Countability and uncountability	18
1.4 Induction	23
1.5 Order in the real line	26
1.6 Some vital inequalities	32
1.7 Exercises	36
<b>2 Metric Spaces</b>	41
2.1 The definition of a metric space	41
2.2 Important metrics in $\mathbb{R}^n$	45
2.3 Open balls and open sets in metric spaces	48
2.4 Closed sets and limit points	53
2.5 Interior, closure, and boundary	59
2.6 Dense subsets	62
2.7 Equivalent metrics	63
2.8 Normed vector spaces	68
2.9 A brief note about conventions	71
2.10 Exercises	71
<b>3 Sequences</b>	77
3.1 Convergence	78
3.2 Discrete dynamical systems	87
3.3 Sequences and limit points	92
3.4 Algebraic theorems for sequences	95
3.5 Subsequences	101
3.6 Completeness	107
3.7 The contraction mapping principle	110
3.8 Sets of sequences as metric spaces	119
3.9 Exercises	125
<b>4 Continuity</b>	131
4.1 The definition of continuity	131

4.2	Equivalent formulations of continuity	142
4.3	Continuity and limit theorems for scalar-valued functions	146
4.4	Continuity and products of metric spaces	150
4.5	Uniform continuity	154
4.6	The metric space $C([a, b], \mathbb{R})$	160
4.7	An application to functional equations	167
4.8	Exercises	170
<b>5</b>	<b>Compactness and Connectedness</b>	177
5.1	Basic definitions and results on compactness	177
5.2	The nested set property for compact sets	181
5.3	Compactness and continuity	183
5.4	Other facts about compactness	185
5.5	Connectedness	191
5.6	Periodic points of maps on intervals	195
5.7	Injective continuous functions defined on intervals	200
5.8	Exercises	202
<b>6</b>	<b>The Derivative</b>	209
6.1	The definition of the derivative	209
6.2	Differentiation rules	218
6.3	Applications of the derivative	222
6.4	Exercises	229
<b>7</b>	<b>The Riemann Integral</b>	235
7.1	Partitions and the definition of the integral	235
7.2	Basic properties of the integral	243
7.3	The fundamental theorem of calculus	247
7.4	Ordinary differential equations	253
7.5	Exercises	257
<b>8</b>	<b>Sequences of Functions</b>	261
8.1	Infinite series	261
8.2	Power series	267
8.3	Higher derivatives and Taylor polynomials	275
8.4	Differentiation and integration of sequences of functions	281
8.5	The exponential function	287
8.6	Compact subsets in $C[a, b]$	290
8.7	Exercises	295
<b>9</b>	<b>Chaos in Discrete Dynamical Systems</b>	301
9.1	The definition of chaos	302
9.2	Semiconjugacy	310
9.3	Subshifts of finite type	314
9.4	Itineraries and piecewise expanding maps	319
9.5	A dynamical system with a dense orbit but no periodic points	331
9.6	Exercises	337

Contents	ix
<b>10 The Hausdorff Metric and Fractals</b>	341
10.1 Definition of the Hausdorff metric	341
10.2 Properties of the Hausdorff metric	343
10.3 Fractals in the plane	346
10.4 Exercises	352
<b>Bibliography</b>	355
<b>Index</b>	357