

Preface

Mathematics is not about calculations, but ideas. My goal as a teacher is to provide students with the opportunity to grapple with these ideas and to be immersed in the process of mathematical discovery. Repeatedly engaging in this process hones the mind and develops mental maturity marked by clear and rigorous thinking. Like music and art, mathematics provides an opportunity for enrichment, experiencing beauty, elegance, and aesthetic value. The medium of a painter is color and shape, whereas the medium of a mathematician is abstract thought. The creative aspect of mathematics is what captivates me and fuels my motivation to keep learning and exploring.

While the content we teach our students is important, it is not enough. An education must prepare individuals to ask and explore questions in contexts that do not yet exist and to be able to tackle problems they have never encountered. It is important that we put these issues front and center and place an explicit focus on students producing, rather than consuming, knowledge. If we truly want our students to be independent, inquisitive, and persistent, then we need to provide them with the means to acquire these skills. Their viability as professionals in the modern workforce depends on their ability to embrace this mindset.

When I started teaching, I mimicked the experiences I had as a student. Because it was all I knew, I lectured. By standard metrics, this seemed to work out just fine. Glowing student and peer evaluations, as well as reoccurring teaching awards, indicated that I was effectively doing my job. People consistently told me that I was an excellent teacher. However, two observations made me reconsider how well I was really doing. Namely, many of my students seemed to depend on me to be successful, and second, they retained only some of what I had taught them. In the words of Dylan Retsek:

Things my students claim that I taught them masterfully, they don't know.

Inspired by a desire to address these concerns, I began transitioning away from direct instruction towards a more student-centered approach. The goals and philosophy behind inquiry-based learning (IBL) resonate deeply with my ideals, which

is why I have embraced this paradigm. According to the Academy of Inquiry-Based Learning, IBL is a method of teaching that engages students in sense-making activities. Students are given tasks requiring them to solve problems, conjecture, experiment, explore, create, and communicate—all those wonderful skills and habits of mind that mathematicians engage in regularly. This book has IBL baked into its core.

This book is intended to be a task sequence for an introduction to proof course that utilizes an IBL approach. The primary objectives of this book are to:

- Expand the mathematical content knowledge of the reader,
- Provide an opportunity for the reader to experience the profound beauty of mathematics,
- Allow the reader to exercise creativity in producing and discovering mathematics,
- Enhance the ability of the reader to be a robust and persistent problem solver.

Ultimately, this is really a book about productive struggle and learning how to learn. Mathematics is simply the vehicle.

You can find the most up-to-date version of this textbook on GitHub:

<http://dcernst.github.io/IBL-IntroToProof/>

I would be thrilled if you used this textbook and improved it. If you make any modifications, you can either make a pull request on GitHub or submit the improvements via email. You are also welcome to fork the source and modify the text for your purposes as long as you maintain the Creative Commons Attribution–Share Alike 4.0 International License.

Much more important than specific mathematical results are the habits of mind used by the people who create those results. ...Although it is necessary to infuse courses and curricula with modern content, what is even more important is to give students the tools they will need in order to use, understand, and even make mathematics that does not yet exist.

Cuoco, Goldenberg, & Mark in *Habit of Mind: An Organizing Principle for Mathematics Curriculum*