Preface

This volume contains papers which friends and colleagues of Friedrich Karpelevich (1927–2000) dedicate to his memory.

Friedrich Karpelevich was born in Moscow, on October 2, 1927. His teenage years coincided with the difficult war time. For several years high schools were closed. Friedrich worked on a factory; as a result of an accident he lost a part of a finger. He was already 20 when he entered the university, having seriously considered the option of foregoing university education. From 1947 to 1952 he was a student at Moscow University. He was one of most brilliant students and started research in his first undergraduate years under the supervision of E. Dynkin. Friedrich participated in Dynkin's seminar for high school students and continued to participate in Dynkin's seminars for a substantial part of his mathematical life (see Dynkin's contribution in this volume). He established himself as a serious mathematician with his paper about the characteristic roots of matrices with nonnegative entries, published in 1949. This paper contains a complete solution of a problem posed by A. Kolmogorov. Before Karpelevich, this problem had been considered under some restrictions by N. Dmitriev and E. Dynkin.

In the early fifties Karpelevich studied subalgebras of semisimple Lie algebras. He started by giving a description of non-semisimple maximal subalgebras of simple complex Lie algebras. The classification of such subalgebras had been obtained earlier by V. Morosov. For the next five years he studied semisimple subalgebras of real semisimple Lie algebras. Shortly before that Dynkin had given a description of the semisimple subalgebras of complex semisimple Lie algebras. The case of real Lie algebras is much more difficult. Here Karpelevich's results include a general statement about a canonical embedding of a real semisimple Lie subalgebra, which became widely used. To obtain this result, Karpelevich applied the theory of symmetric spaces, which became his favorite mathematical subject. To complete the study for the case of classical Lie algebras he had to work on very complex problems of linear algebra. He got a remarkable formula for the inertia index of an invariant symmetric or Hermitian form on the space of irreducible representation of the real semisimple Lie algebra, which solves the problem completely. These results comprised Friedrich's PhD thesis, and they were rewarded in 1956 by a very prestigious Moscow Mathematical Society Prize for Young Mathematicians. A. Onishchik prepared for this volume an expository paper about this work.

Due to anti-Semitism, Karpelevich was not admitted to graduate school at Moscow University. He was preparing his thesis while teaching in a provincial technical school in Novocherkassk and, starting in 1953, in the Moscow Institute of

Transport Engineering. Karpelevich worked in this Institute up to last days of his life.

At the end of the fifties Friedrich's interests shifted to geometry and analysis on homogeneous manifolds. Together with F. Berezin in 1958, he computed zonal spherical functions on Grassmannians in terms of special functions of one variable. If the space is of rank one, zonal functions are functions of one variable and can be expressed in terms of the Gauss hypergeometric function. For complex classical groups they were computed by Gelfand and Naimark. Numerous attempts to do it in other cases were unsuccessful, so the computation made by Berezin and Karpelevich remains unique up to this day.

The natural development of Friedrich's interest in spherical functions was our collaboration on the computation of the c-function of Harish-Chandra in 1962, and later on the inverse horospherical transform. You can find more details about this work in my reminiscences in this volume.

One of the most important of Karpelevich's results in the theory of symmetric spaces is his construction of the boundary of symmetric spaces of non-positive curvature in 1965. It is based on a detailed study of the asymptotic behavior of the geodesics. Karpelevich's boundary has numerous applications in the theory of the eigenfunctions of the Laplace–Beltrami operator, which he studied for some time. Soon he changed his field and started to work in probability theory. Yu. Neretin wrote an expository paper on boundaries of symmetric spaces for this collection. Karpelevich's works in probability are reflected in the memorial volume "Analytic methods in applied probability", Yu. M. Suhov (ed.), American Mathematical Society, Providence, RI, 2002.

Friedrich Karpelevich was one of the deepest and most original mathematicians working in Lie groups and symmetric spaces in the second half of the 20th century. The contributors of this volume have different relationships with him. Some of them were happy to know Friedrich personally and to collaborate with him, some know him only through his works, but they all share the highest opinion about Karpelevich's mathematical merits.

As we already mentioned, the volume includes Dynkin's recollection on Karpelevich's first steps in mathematics and two expository papers on Karpelevich's results and their development: Onishchik's paper on subalgebras of real semisimple Lie algebras and Neretin's paper on boundaries of symmetric spaces. Several papers are connected with the product formula for the Harish-Chandra c-function. A. Knapp discusses its applications to intertwining operators. The papers by Oshima and by Krötz and Ólafsson consider computations of c-functions for some pseudo-Riemannian symmetric spaces. I recall that Friedrich had a strong interest in geometry and analysis on pseudo-Riemannian symmetric spaces and undertook several attempts to work in this direction. I wrote down my reminiscences on our joint work on the c-function and the horospherical transform.

The paper of Hilgert, Pasquale and Vinberg considers an algebraic version of the inversion of the horospherical transform which is also connected with the c-function.

Several papers are dedicated to Karpelevich's favorite area, symmetric spaces. Sawyer gave a survey of the Abel transform on noncompact Riemann symmetric spaces which has strong connections with the horospherical transform and its inversion. It also has connections with estimates of the heat kernel on such spaces. Anker and Ostellari prepared the survey on this area. In Faraut's contribution, some analytic problems of complex crowns of Riemann symmetric spaces are studied.

The volume contains several contributions on theory of representations and other aspects of Lie groups: Kobayashi and Nasrin's paper on a new multiplicity one theorem, Molchanov's paper on canonical representations on Hermitian symmetric spaces, and Baruch, Piatetski-Shapiro and Rallis' paper on representations of $U_{2,1}$ for local fields. Akhiezer investigates asymptotic problems for group rings of compact Lie groups.

Alekseevsky and Di Scala generalize Karpelevich's 1953 result on totally geodesic orbits of reductive isometry groups on Riemann symmetric spaces. The volume also contains a paper by Macdonald on affine root systems, a paper by Gelfand, Retakh, and Wilson on quaternionic determinants and quasideterminants, a contribution by Olshanetsky and Rogov on non-commutative hyperbolic spaces, a paper by Enriquez and Etingof on quantization of dynamical r-matrices, and a paper by Bernstein and Gindikin on integral geometry for curves.

We can see that many of these contributions have a close relation with Karpelevich's heritage and all of them, without a single exception, represent the areas of mathematics which he loved all his life.

Simon Gindikin March 2003

Papers of F. Karpelevich on Lie Groups and Symmetric Spaces

1. F. I. Karpelevich, *On nonsemisimple maximal subalgebras of semisimple Lie Algebras*, Doklady Akad. Nauk SSSR **76** (1951), 775–778. (Russian)
2. ______, *Classification of simple subgroups of the real forms of the group of complex unimodular matrices*, Doklady Akad. Nauk SSSR **85** (1952), 1205–1208. (Russian)
3. ______, *Subgroups of real Lie groups*, Uspekhi Mat. Nauk **7** (1952), no. 5, 203–204. (Russian)
4. ______, *Surfaces of transitivity of a semisimple subgroup of the group of shifts of a symmettric space*, Doklady Akad. Nauk SSSR **93** (1953), 401–404. (Russian)
5. ______, *The classification of the simple subalgebras of the real forms of classical Lie algebras*, Doklady Akad. Nauk SSSR **93** (1953), 613–616. (Russian)
6. ______, *Simple subalgebras of the real Lie algebras*, Trudy Moskov. Mat. Obshch. **4** (1955), 3-112. (Russian)
7. ______, *On semisimple subgroups of semisimple Lie groups*, Uspekhi Mat. Nauk **10** (1955), no. 1, 196. (Russian)
8. ______, *Hermitian and bilinear invariants of subalgebras of the matrix algebra*, Uspekhi Mat. Nauk **10** (1955), no. 4, 190–191. (Russian)
9. F. I. Karpelevich and A. L. Onishchik, *Homology algebra of the path space*, Doklady Akad. Nauk SSSR **106** (1956), 967–969. (Russian)
10. F. I. Karpelevich, *On the fibering of homogeneous spaces*, Uspekhi Mat. Nauk **11** (1956), no. 3, 131-138. (Russian)
11. F. A. Berezin and F. I. Karpelevich, *Zonal spherical functions and Laplace operators on some symmetric spaces*, Doklady Akad. Nauk SSSR **118** (1958), 9–12. (Russian)
12. F. I. Karpelevich, V. N. Tutubalin, and M. G. Shur, *Limit theorems for compositions of distributions on the Lobachevski plane and space*, Teor. Veroyatnost. i Primenen. **4** (1959), 432–436; English transl. in Theory Probab. Appl, **4** (1959).
13. F. I. Karpelevich, *Geodesics and harmonic functions on symmetric spaces*, Doklady Akad. Nauk SSSR **124** (1959), 1199–1202. (Russian)
14. ______, *Horospherical radial parts of the Laplace operator on symmetric spaces*, Doklady Akad. Nauk SSSR **143** (1962), 1034–1037; English transl., Soviet Math. Dokl. **3** (1962), 528–531.
15. S. G. Gindikin and F. I. Karpelevich, *Plancherel measure for Riemannian symmetric spaces of nonpositive curvature*, Doklady Akad. Nauk SSSR **145** (1962), 252–255; English transl., Soviet Math. Dokl. **3** (1962), 962–965.

16. F. I. Karpelevich, *Nonnegative eigenfunctions of the Beltrami-Laplace operator on symmetric spaces of nonpositive curvature*, Doklady Akad. Nauk SSSR **151** (1963), 1274–1276; English transl., Soviet Math. Dokl. **4** (1963), 1180–1182.
17. M. I. Graev, F. I. Karpelevich, and A. A. Kirillov, *Represenration theory of the Lie groups*, Proc. Fourth All-Union Math. Congress (Leningrad, 1961), vol. II, Nauka, Leningrad, 1964, pp. 275–281. (Russian)
18. S. G. Gindikin and F. I. Karpelevich, *A problem of integral geometry*, In Memoriam: N. G. Chebotarev, Izdat. Kazan. Univ, Kazan, 1964, pp. 30–43; English transl., Selecta Math. Sovietica **1** (1981), 169–184.
19. F. I. Karpelevich, *Geometry of geodesics and eigenfunctions of the Beltrami-Laplace operator on symmetric spaces*, Trudy Moskov. Mat. Obshch. **14** (1965), 48–185; English transl., Trans. Moscow Math. Soc. **1965** (1967), 51–199.
20. S. G. Gindikin and F. I. Karpelevich, *On certain special functions of several variables related with Lie groups*, Studies Contemporary Problems Constructive Theory of Functions, Izd. Akad. Nauk Azerb. SSR, Baku, 1965, pp. 545–554. (Russian)
21. ——, *An integral associated with Riemannian symmetric spaces of nonpositive curvature*, Izv. Akad. Nauk SSSR Ser. Mat. **30** (1966), 1148–1156; English transl., Amer. Math. Soc. Transl. (2) **85** (1969), 249–257.
22. F. A. Berezin and F. I. Karpelevich, *On associative algebras of functions*, Vestnik Moskov. Univ. Ser. I Mat. Mekh. **1976**, no. 1, 33–38; English transl., Moscow Univ. Math. Bull. **31** (1976), no. 1, 29–34.