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Introduction

In these notes we present the first steps of a *theory of confoliations* which is designed to link the geometry and topology of 3-dimensional contact structures with the geometry and topology of codimension 1 foliations on 3-manifolds. A confoliation is a mixed structure which interpolates between a codimension 1 foliation and a contact structure. This object (without the name) first appeared in work of Steve Altschuler (see [**3**]). In this paper we will be concerned exclusively with the 3-dimensional case, although confoliations can be defined on higher dimensional manifolds as well (see Section 1.1.6 below).

Foliations and contact structures have been studied practically independently. Indeed at the first glance, these objects belong to two different worlds. The theory of foliations is a part of topology and dynamical systems while contact geometry is an odd-dimensional brother of symplectic geometry. However, the two theories have developed a number of striking similarities. In each case, an understanding developed that additional restrictions are important on foliations and contact structures to make them interesting and useful for applications, for otherwise, these structures are so flexible that they fit anywhere, hence produce no information about the topology of the underlying manifold. This extra flexibility is caused by the appearance of *Reeb components* in the case of foliations (see [**48**], [**56**], [**54**]), and by the *overtwisting phenomenon* in the case of contact structures (see [**7**], [**11**]). In order to make the structures more rigid in the context of foliations the theory of *taut foliations* was developed ([**53**], [**52**] [**20**]), as well as the related theory of essential laminations ([**21**], [**32**]). In the parallel world of contact geometry, the theory of *tight contact structures* were developed for similar purposes ([**7**], [**12**], [**24**]).

The theory of confoliations should help us to better understand links between the two theories and should provide an instrument for transporting the results from one field to the other.

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