

Preface

In the spring of 2002 I gave a series of graduate lectures at Penn State on ‘coarse geometry’. These are the edited lecture notes from that course. The intention was to discuss various aspects of the theory of ‘large scale structures’ on spaces, with a particular focus on the notions of asymptotic dimension and uniform embeddability into Hilbert space, which have recently proved of significance for the Novikov conjecture. On the other hand, so far as is consistent with the preceding objective, the study of C^* -algebras arising from coarse geometry has been de-emphasized; this has already been written about at length elsewhere [59].

The first few chapters of the book are devoted to a general perspective on ‘coarse structures’ which was first set out in the paper [34]. This notion has the advantage of including under one heading many of the different notions of ‘control’ that have been used by topologists (for example [2]); and even when only metric coarse structures are in view, the abstract framework brings the same simplification as does the passage from epsilons and deltas to open sets when speaking of continuity. In this more general context one can still discuss ideas like growth, amenability, and coarse cohomology, and these are addressed in chapters 3 through 5.

The middle section of the notes reviews notions of negative curvature and rigidity. Modern interest in large scale geometry derives in large part from Mostow’s rigidity theorem, with its crucial insight that the coarse structure of hyperbolic space determines the quasiconformal structure of the boundary, and from Gromov’s subsequent ‘large scale’ rendition of the crucial properties of negatively curved spaces. There are many excellent expositions of this material and our account is brief in places.

In the final sections we discuss recent results on asymptotic dimension (mostly due to Bell and Dranishnikov) and uniform embedding into Hilbert space. We also take the opportunity to review the beautiful construction of Skandalis, Tu and Yu [62] which allows one to encode the large scale structure of a (bounded geometry) space by means of a suitable groupoid.

The large scale geometry of *discrete groups* is a beautiful and active area of research, and in these notes we barely scratch its surface. The reader who wants to learn more about geometric group theory is referred to the books [13, 25, 17].

I am grateful to the National Science Foundation for their support under grants DMS-9800765 and DMS-0100464.

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