

## Contents

Preface	vii
Chapter 1. Introduction to $q$ -Analogues and Symmetric Functions	1
Permutation Statistics and Gaussian Polynomials	1
The Catalan Numbers and Dyck Paths	6
The $q$ -Vandermonde Convolution	8
Symmetric Functions	10
The RSK Algorithm	17
Representation Theory	22
Chapter 2. Macdonald Polynomials and the Space of Diagonal Harmonics	27
Kadell and Macdonald's Generalizations of Selberg's Integral	27
The $q, t$ -Kostka Polynomials	30
The Garsia-Haiman Modules and the $n!$ -Conjecture	33
The Space of Diagonal Harmonics	35
The Nabla Operator	37
Chapter 3. The $q, t$ -Catalan Numbers	41
The Bounce Statistic	41
Plethystic Formulas for the $q, t$ -Catalan	44
The Special Values $t = 1$ and $t = 1/q$	47
The Symmetry Problem and the $\text{dinv}$ Statistic	48
$q$ -Lagrange Inversion	52
Chapter 4. The $q, t$ -Schröder Polynomial	59
The Schröder Bounce and Area Statistics	59
Recurrences and Explicit Formulae	62
The Special Value $t = 1/q$	65
The Delta Operator	68
The Schröder $\text{dinv}$ Statistic	70
The Limit as $d \rightarrow \infty$	73
Chapter 5. Parking Functions and the Hilbert Series	77
Extension of the $\text{dinv}$ Statistic	77
An Explicit Formula	79
The Statistic $\text{area}'$	82
The $\text{pmaj}$ Statistic	82
The Cyclic-Shift Operation	85
Chapter 6. The Shuffle Conjecture	91

A Combinatorial Formula for the Character of the Space of Diagonal Harmonics	91
Path Symmetric Functions and LLT Polynomials	92
Superization	99
The Fermionic Formula	101
Skewing, Balanced Paths, and Hall-Littlewood Polynomials	103
The $m$ -parameter	107
Chapter 7. The Proof of the $q, t$ -Schröder Theorem	113
Summation Formulas for Generalized Skew and Pieri Coefficients	113
The Proof	119
Some Related Results	120
Appendix A. The Combinatorics of Macdonald Polynomials	123
The Monomial Statistics	123
Proof of the Formula	125
Consequences of the Formula	129
Nonsymmetric Macdonald Polynomials	135
The Genesis of the Macdonald Statistics	139
Appendix B. The Loehr-Warrington Conjecture	141
The Conjecture	141
Expansion into LLT polynomials	144
Appendix C. Solutions to Exercises	147
Chapter 1	147
Chapter 2	149
Chapter 3	152
Chapter 4	154
Chapter 5	156
Chapter 6	157
Chapter 7	159
Appendix A	160
Appendix B	162
Bibliography	163