Preface

The geometry of the domains of holomorphic and CR functions is one of the primary topics in contemporary mathematics. It is a field where various techniques converge: PDE's, real/complex vector fields, induced systems, boundary values, analytic discs in symplectic spaces, just to mention a few. The excellent interplay of these tools serves as a model both for research and education. This book collects my lectures addressed to graduate students at the University of Padova during the last three years as well as an introduction to part of my present research. The purpose is twofold: to provide a basic foundation in the theory of several complex variables and CR functions and to give some new insight and tools in current research.

Chapter 1 introduces the theory of complex analytic functions. It starts with classical facts about power series and plurisubharmonic functions along the guidelines of Oka and Hörmander. Next, it begins the discussion of the equivalence between pseudoconvex domains and domains of holomorphy and proves by geometric evidence the coincidence of the pseudoconvexity of a domain with that of its boundary. Finally, it solves the $\bar\partial$-Neumann problem by the method of Kohn. This implies the solution of the Levi problem and provides the complete proof of the equivalence mentioned above. It also includes some new results concerning the regularity of $\bar\partial$ at a weakly q-pseudoconvex boundary.

Chapter 2 deals with real structures. Along with the classical Frobenius theorem, it presents its symplectic counterpart, that is, the Darboux-Frobenius theorem. Finally, it discusses subellipticity and hypoellipticity of systems of real vector fields.

The content of Chapter 3 is CR structures. Particular emphasis is devoted to the analysis of the conormal bundle to a real submanifold of $\mathbb{C}^n$ under a canonical transformation. This serves to describe Hamiltonian/Levi foliations and also to interchange a higher-codimensional submanifold with a hypersurface. Next, it recalls the theory of analytic discs attached to real submanifolds and of their infinitesimal deformations. Furthermore, it introduces a new family of discs with singular boundary. They yield a new proof and a refined statement of the classical results on the extension of CR functions from manifolds of finite type. The problem of the construction of lifts and partial lifts of analytic discs is also addressed: the discussion fully discloses the symplectic-geometric character of these objects. By means of them, there is obtained in a natural way a "connection" on the conormal bundle to a real submanifold of $\mathbb{C}^n$. The coupling between lifts and infinitesimal deformations is then exploited. It first shows that CR extendibility evolves along a CR curve according to the dual-inverse connection of the above one. Also, when M is "minimal" in the sense of Tumanov, it yields the wedge extension of CR functions. The discussion ends with the theory of separate real analyticity, the real counterpart of the Hartogs theorem, which takes a geometric explanation here.

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