

Contents

Preface	ix
1. Introduction	1
2. Basic results in classical minimal surface theory	13
2.1. Eight equivalent definitions of minimality	13
2.2. Weierstrass representation	17
2.3. Minimal surfaces of finite total curvature	19
2.4. Periodic minimal surfaces	20
2.5. Some interesting examples of complete minimal surfaces	22
2.6. Monotonicity formula and classical maximum principles	33
2.7. Ends of properly embedded minimal surfaces	35
2.8. Second variation of area, index of stability and Jacobi functions	37
2.9. Barrier constructions	41
3. Minimal surfaces with finite topology and more than one end	45
3.1. Classification results for embedded minimal surfaces of finite total curvature	45
3.2. Constructing embedded minimal surfaces of finite total curvature	48
4. Limits of embedded minimal surfaces without local area or curvature bounds	53
4.1. Colding-Minicozzi theory	53

4.1.1. Main strategy of the proof of Theorem 4.1.3.	58
4.1.2. More details on the main strategy	59
4.2. Minimal laminations with isolated singularities	69
5. The structure of minimal laminations of \mathbb{R}^3	73
6. The Ordering Theorem for the space of ends	77
7. Conformal structure of minimal surfaces	81
7.1. Recurrence and parabolicity for manifolds	81
7.2. Universal superharmonic functions	84
7.3. Quadratic area growth and middle ends	86
8. Uniqueness of the helicoid I: Proper case	91
9. Embedded minimal annular ends with infinite total curvature	95
9.1. Harmonic functions on annuli	95
9.2. Annular minimal ends of infinite total curvature	97
10. The embedded Calabi-Yau problem	103
10.1. Uniqueness of the helicoid II: Complete case	103
10.2. Regularity of the singular set of convergence of a limit minimal lamination	106
11. Local pictures, local removable singularities and dynamics	113
12. Embedded minimal surfaces of finite genus	123
12.1. The Hoffman-Meeks conjecture	123
12.2. Non-existence of one-limit-ended examples	125
12.3. Uniqueness of the Riemann minimal examples	128
12.4. Colding-Minicozzi theory (fixed genus)	133
13. Topological aspects of minimal surfaces	137
14. Partial results on the Liouville Conjecture	145
15. The Scherk Uniqueness Theorem	149
16. Calabi-Yau problems	153
17. Outstanding problems and conjectures	157
Bibliography	171