Preface

This book explains some recent progress in combinatorial geometry that comes from an unexpected connection with polynomials and algebraic geometry. One of the early results in this story is a two-page solution of a problem called the finite field Kakeya problem, which experts had believed was extremely deep. The most well-known result in this book is an essentially sharp estimate for the distinct distance problem in the plane, a famous problem raised by Paul Erdős in the 1940s. The book also emphasizes connections between different fields of mathematics. For example, some of the new proofs in combinatorics that we study were suggested by ideas from error-correcting codes. We discuss this connection, as well as related ideas in Fourier analysis, number theory, and differential geometry. First- or second-year graduate students, as well as advanced undergraduates and researchers, should find this book accessible.

My own work in this area is mostly joint with Nets Katz, and I learned a lot about this circle of ideas talking with him and exploring together. I taught a class on this material at MIT in the fall of 2012. I want to thank the students in the class who typed up notes for some of the lectures. Those lecture notes formed a first draft for the book. The students were Sam Elder, Andrey Grinshpun, Nate Harmon, Adam Hesterberg, Chiheon Kim, Gaku Liu, Laszlo Lovasz, Rik Sengupta, Efrat Shaposhnik, Sean Simmons, Yi Sun, Adrian Vladu, Ben Yang, and Yufei Zhao. I also want to thank the following people for looking at drafts of the book and making helpful suggestions: Josh Zahl, Thao Do, Hong Wang, Ben Yang, and Jiri Matoušek. While I was writing the book, I was supported by a Sloan fellowship and a Simons Investigator award.

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