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Preface

We celebrate the fiftieth anniversary of one of the most classical models in probability theory. In this book, we describe the main results of first-passage percolation, paying special attention to the recent burst of advances. The purpose of this book is twofold. We first give self-contained proofs of seminal results obtained in the '80s and '90s on limit shapes and geodesics, while covering the state of the art of these questions. Second, aside from these classical results, we discuss recent perspectives and directions including (1) the connection between Busemann functions and geodesics, (2) proofs of sublinear variance of the passage time, and (3) the role of growth and competition models. We also provide a collection of old and new open questions, in the hope that they will be solved before the one hundredth anniversary.

