Preface

Batalin-Vilkovisky ("BV") formalism arose in the beginning of the 1980s as a tool of mathematical physics designed to define the path integral for gauge theories. Since then the construction turned out to be very useful for applications in algebraic topology—invariants of 3-manifolds and knots, Chas-Sullivan string topology, operations on rational cohomology of CW complexes. Another spectacular application of the BV formalism is Kontsevich's deformation quantization of Poisson manifolds. The general direction these applications go in is via applying BV formalism to define the path integral for particular models of topological field theory and then finding an appropriate interpretation for the value of the path integral (and proving the desired properties).

These lectures were given at the University of Notre Dame in the fall of 2016 for a graduate mathematical audience; a previous iteration of this course was given in the fall of 2014 at the Max Planck Institute for Mathematics, Bonn, jointly with the University of Bonn. The aim of these courses was to give an introduction to the perturbative path integral for gauge theories (in particular, topological field theories) in Batalin-Vilkovisky formalism and some of its applications. The courses were oriented toward a mathematical audience and did not require any prior physics background. To elucidate the picture, we were mostly focusing on finite-dimensional models for gauge systems and path integrals, while giving comments on what has to be amended in the infinite-dimensional case relevant to local field theory. Our motivating examples included Alexandrov-Kontsevich-Schwarz-Zaboronsky sigma models; the perturbative expansion for Chern-Simons invariants of 3-manifolds, given in terms of integrals over configurations of points on the manifold; the BF theory on cellular decompositions of manifolds; and the Kontsevich's deformation quantization formula.

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