Preface

This book is about combinatorial properties of convex sets, of families of convex sets in d-dimensional spaces, and properties of finite points sets in $\mathbb{R}^d$ related to convexity. Typical examples are the classical theorems of Helly, Carathéodory, Radon, and Erdős-Szekeres. More recent results of the same type are Tverberg's theorem, the fractional Helly theorem, the colourful Carathéodory theorem, and the (p, q) theorem of Alon and Kleitman.

The book is based on series of lectures I gave at various places, including the Eötvös University in Budapest, KAIST in Daejeon, South Korea, UPC in Barcelona, and IHP in Paris. The audience consisted of students, graduate and undergraduate alike, postdocs, and research mathematicians. I have benefitted from their questions and reactions and also from the notes that were written following the lectures at IHP. I used those notes for this book and my thanks go to the students who wrote them.

The title is *Combinatorial Convexity*, which tries to reflect that the topic is convexity and also geometry, and that it is close to discrete mathematics. The questions considered in this book are frequently of a combinatorial nature and the proofs use ideas from geometry and are often combined with graph and hypergraph theory. Sometimes the proof is completely combinatorial but uses a little help from geometry in the form of a simple (or not so simple) lemma. In the text I have tried to highlight these geometric ideas whenever they are used.

Not much background is needed. Most important are basic linear algebra (including the separation theorem and linear programming) and some graph and hypergraph theory. The reader is assumed to be familiar with asymptotic estimations and the big O notation. The first chapter in the book describes the necessary prerequisites. Let me repeat: not much background is needed, although some mathematical maturity is expected from the imaginary reader.

The book is intended for students (graduate and undergraduate alike), but postdocs and interested research mathematicians might also find it useful. It can be used as a textbook. The chapters are typically short, each suitable for a one- or two-hour lecture. At the end of each chapter there are some exercises of various levels of difficulty. Solving them would help the reader to better understand the material. There is hardly any new material in the book; the exception is Theorem 22.2, and the treatment of a stretched grid in Chapters 20 and 27 is different from the usual one.

I want to emphasize that the material in this book is quite up-to-date, yet the results stated are often not the strongest available. In such cases a short description and/or a pointer are given to help the reader locate the current best result. I have avoided several parts of discrete geometry that do not fit into my image of combinatorial convexity. Some examples are algorithmic issues, computational geometry, packings and coverings, and abstract convexity. This book is too short to contain them. Here is a list of other monographs and surveys that cover similar topics:

L. Danzer, B. Grünbaum, V. Klee: *Helly's theorem and its relatives* (1963),

J. Eckhoff: *Helly, Radon, and Carathéodory type theorems* (1993),

J. Pach and P. Agarwal: *Combinatorial geometry* (1995),

J. Matoušek: *Lectures on discrete geometry* (2002).

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When writing such a book, typos and errors are almost completely unavoidable. My counting shows that I normally leave about 10-15 such mistakes per page on the first writing. Most of them are discovered on the first or second reading by myself. But there always remain some obstinate ones that are hard to detect. Readers are welcome to inform me of typos and errors that I will be able to correct in a new edition or in the e-version of this book.

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