I say, however, that a monogenic function of $x + iy$ can be formed by aid of *any* two solutions of Laplace's equation and without any quadrature. Suppose P and Q to be any two such solutions; they do not then in general satisfy equations (1). If they do satisfy (1), then $P + iQ$ is the function sought. If they do not satisfy (1), write

$$Q_1 = \frac{\partial P}{\partial x} - \frac{\partial Q}{\partial y},$$

$$P_1 = \frac{\partial P}{\partial y} + \frac{\partial Q}{\partial x}.$$

Then $P_1 + iQ_1$ is a monogenic function of $x + iy$, for

$$\frac{\partial P_1}{\partial x} - \frac{\partial Q_1}{\partial y} = 0,$$

$$\frac{\partial P_1}{\partial y} + \frac{\partial Q_1}{\partial x} = 0,$$

$$\nabla P_1 = \nabla Q_1 = 0,$$

$$\text{since } \nabla P = \nabla Q = 0.$$

LAMBERT'S NON-EUCLIDEAN GEOMETRY.

BY PROF. GEORGE BRUCE HALSTED.

IN the discussion which followed my lecture in the Mathematical Section of the Congress at Chicago, Professor Study of Marburg mentioned that there had recently been brought to light an old paper of Lambert's on what was long after named the non-Euclidean geometry. Professor Klein jotted down on my Lobatschewsky programme the address of Dr. Staeckel, as the person from whom I might hope for definite information; and from his answer to my letter I extract the following highly interesting facts.

This essay of Lambert's bears the title: "Zur Theorie der Parallellinien." It is dated September, 1766, but was first published in 1786 from the papers left by F. Bernoulli, a relative of John Bernoulli. It appeared in the

"*Leipziger Magazin für reine und angewandte Mathematik;* herausgegeben von J. Bernoulli und C. F. Hindenburg, erster Jahrgang 1786. S. 13 ff." On account of the extraordinary interest of this article and the present great rarity of this magazine, the Leipziger Gesellschaft der Wissenschaften, at the suggestion of Dr. Staeckel, will reprint it in their *Abhandlungen.* Meanwhile, to show how wonderfully it anticipates Lobatschewsky, Bolyai, Riemann, Beltrami, in what it maintains, we need only cite the following:

(1) The parallel-axiom needs proof, since it does not hold for geometry on the surface of the sphere.

(2) In order to make intuitive a geometry in which the triangle's angle-sum is less than two right angles we need an "imaginary" sphere [pseudo-sphere].

(3) In a space in which the triangle's angle-sum is different from two right angles, there is an absolute measure [a natural unit for length].

The whole paper is another unexpected illustration of the words in a letter from Sir Robert Ball: "It is also noteworthy how many mathematicians, approaching the subject from very varied sides, have been led to the study of what mathematics would be like without the eleventh axiom."

AUSTIN, TEXAS, *November*, 1893.

THE TEACHING OF MATHEMATICS AT GÖTTINGEN.*

THE purpose of the following remarks is not to furnish to students who wish to prepare themselves to teach mathematics and physics in the higher schools, a detailed scheme of the lectures and exercises which they should attend during each semester. It would be impossible to do this on account of the great number of branches of mathematics and mathematical physics and the frequent changes necessary in the subjects and the arrangement of the courses of lectures. It is essential, however, that students should be acquainted with

* Translation of a circular, "Announcement of a scheme of study for those wishing to become teachers of mathematics and physics; together with an extract from the regulations of the seminary of mathematics and physics," issued by the University, and reprinted in the *Zeitschrift für math. und naturwiss. Unterricht*, vol. 24. pp. 540-546. Apart from its relation to the work at Göttingen, the circular i of general interest to teachers of mathematical science.—T. S. F.